Division E

Mathematical Olympiads for Elementary & Middle Schools

November 12, 2013

Contest 1

1A *Time: 3 minutes*

What is the value of the product $5 \times 4 \times 5 \times 4 \times 5 \times 4 \times 5$?

1B *Time: 4 minutes*

Madison has five stickers in a row on a piece of paper. The star is one to the left of the puppy. The rainbow is to the right of the heart. The puppy is three to the left of the kitten. Which sticker is in the middle of the row?

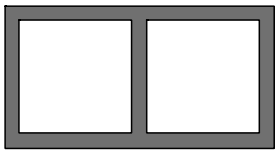
1C *Time: 5 minutes*

For a certain 3-digit number:
- the digits are in increasing order
- the difference of the greatest and least digits is 7
- it is a multiple of 9 and greater than 200.

Find the 3-digit number.

1D *Time: 6 minutes*

Two square gardens are each 10m by 10m. They are enclosed by a sidewalk of width 1m. There is also a shared sidewalk of width 1m between gardens (as shown). Find the total area, in square meters, of the sidewalks.



1E *Time: 7 minutes*

In the multiplication problem at the right, each letter represents a different digit. What 4-digit number is represented by MATH?

$$\begin{array}{r} \text{MATH} \\ \times\ 4 \\ \hline \text{HTAM} \end{array}$$

Please fold over on line. Write answers on back.

Division **E**

Mathematical Olympiads for Elementary & Middle Schools

November 12, 2013

Contest **1**

1A *Student Name and Answer*

1B *Student Name and Answer*

1C *Student Name and Answer*

1D *Student Name and Answer*

sq m

1E *Student Name and Answer*

Please fold over on line. Write answers in these boxes.

2A *Time: 3 minutes*

Gregor is the first person standing on line. His friend, Harold, is the 17th person on that same line. How many people are standing between them on line?

2B *Time: 4 minutes*

How many 2-digit odd numbers are greater than 30?

2C *Time: 5 minutes*

The sum of five numbers divided by 5 is 10. A sixth number is added to the sum of the original five numbers. This sum divided by 6 is 9. What is the sixth number?

2D *Time: 7 minutes*

Each student in an art class has 48 identical one-inch cubes. Each student glues all of his or her cubes together to make a rectangular solid. No two solids are identical. What is the maximum number of students in that art class?

2E *Time: 7 minutes*

Emily plays a game that uses a marker, a coin and a number line. Her marker begins at zero on the number line. She flips the coin. If the coin lands heads up, she moves her marker 3 units to the right. If the coin lands tails up, she moves her marker 10 units to the right. Therefore there are some numbers that the marker cannot land on, such as 1, 2, 4 and 5. What is the greatest whole number on the number line that cannot be landed on?

Please fold over on line. Write answers on back.

Division **E**

Mathematical Olympiads for Elementary & Middle Schools

December 10, 2013

Contest **2**

2A *Student Name and Answer*

2B *Student Name and Answer*

2C *Student Name and Answer*

2D *Student Name and Answer*

2E *Student Name and Answer*

Please fold over on line. Write answers in these boxes.

Division E

Mathematical Olympiads for Elementary & Middle Schools

January 14, 2014

Contest 3

3A *Time: 3 minutes*

What number does N represent in the equation $6 \times 7 \times 40 = 5 \times 8 \times N$?

3B *Time: 4 minutes*

There are 8 marks evenly spaced from each other along a meter stick. The first mark is at 27 cm. The eighth mark is at 62 cm. Where, in centimeters, is the third mark?

3C *Time: 5 minutes*

The numbers 541A3, 541B3, and 541C3 are all different multiples of 3. The letters A, B, and C represent digits within the numbers. What is the product $A \times B \times C$?

3D *Time: 6 minutes*

The sum of the page numbers of Chapter 3 (of a certain book) is 374. If there are 11 pages in Chapter 3, on what page does Chapter 3 begin?

3E *Time: 7 minutes*

Each letter represents a different digit.

What 4-digit number does NEAR represent?

$$\begin{array}{cccccc} & T & E & N & T \\ + & & E & N & T \\ \hline R & A & R & E & S \end{array}$$

Please fold over on line. Write answers on back.

Division **E**

Mathematical Olympiads for Elementary & Middle Schools

January 14, 2014

Contest **3**

3A *Student Name and Answer*

3B *Student Name and Answer*

cm

3C *Student Name and Answer*

3D *Student Name and Answer*

3E *Student Name and Answer*

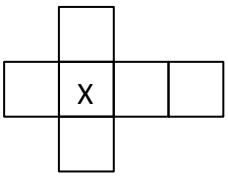
Please fold over on line. Write answers in these boxes.

4A *Time: 3 minutes*

Evaluate: $10 \times 234 + 10 \times 342 + 10 \times 423$

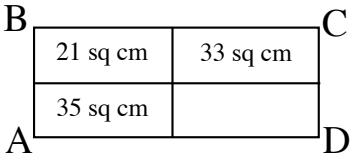
4B *Time: 4 minutes*

In the grid shown, the numbers 1, 2, 3, 4, 5, and 6 are to be placed, one per square. The sum of the numbers in the row going across is 11. The sum of the numbers in the column going down is also 11. What is the number in the box with the X?



4C *Time: 5 minutes*

The areas of three small rectangles are shown in the diagram. The four small rectangles make up a larger, fifth rectangle, *ABCD*. If every measurement is a whole number, how many centimeters is the perimeter of the rectangle *ABCD*?

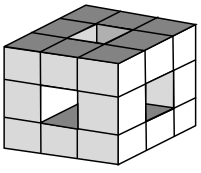


4D *Time: 6 minutes*

How many different three-digit numbers of the form *A*5*B* are divisible by 9? Note that *A* and *B* could be the same digit.

4E *Time: 7 minutes*

A 3x3x3 cube is built out of twenty-seven 1x1x1 blocks. Then, seven 1x1x1 blocks are removed. Specifically, the center block from each of the six faces of the cube is removed and the center block of the cube is removed. What is the total surface area of the modified cube?



Please fold over on line. Write answers on back.

Division **E**

Mathematical Olympiads for Elementary & Middle Schools

February 11, 2014

Contest **4**

4A *Student Name and Answer*

4B *Student Name and Answer*

4C *Student Name and Answer*

4D *Student Name and Answer*

4E *Student Name and Answer*

Please fold over on line. Write answers in these boxes.

5A *Time: 3 minutes*

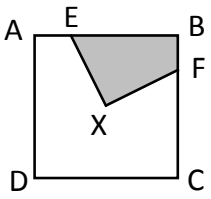
Julio has 23 pencils. He puts the pencils into six boxes. Each box has at least one pencil. There is a different number of pencils in each box. He puts as many pencils as he can into the last box. How many pencils are in the last box?

5B *Time: 4 minutes*

Find the value of *N* for which $\frac{1}{5} = \frac{1}{6} + \frac{1}{N}$.

5C *Time: 6 minutes*

E is one-quarter of the way from A to B. F is one-quarter of the way from B to C. ABCD is a square with an area of 64 and has center X. What is the area of the shaded region (EXFB)?



5D *Time: 6 minutes*

Person A has some number of pieces of candy.
Person B has exactly one-half as many as Person A.
Person C has exactly one-third as many as Person A.
Person D has exactly one-quarter as many as Person A.
If D has at least 4 pieces of candy, what is the fewest number of pieces of candy that the four have all together?

5E *Time: 6 minutes*

In the Marble Club, the average number of marbles the members have is 100. When Person X joins the club with his 80 marbles, the average number of marbles per member becomes 96. How many people, including Person X, are in the Marble Club?

Please fold over on line. Write answers on back.

Division **E**

Mathematical Olympiads for Elementary & Middle Schools

March 11, 2014

Contest **5**

5A *Student Name and Answer*

5B *Student Name and Answer*

5C *Student Name and Answer*

5D *Student Name and Answer*

5E *Student Name and Answer*

Please fold over on line. Write answers in these boxes.

Division E
Mathematical Olympiads for Elementary & Middle Schools
November 12, 2013
Contest 1

SOLUTIONS AND ANSWERS

1A

40,000

1A *Strategy: Regroup the factors to simplify the multiplication.*

Since $5 \times 4 \times 5 = 100$, look for another group of factors that make 100, and then see what factors are left.

$5 \times 4 \times 5 \times 4 \times 5 \times 4 \times 5 = (5 \times 5 \times 4) \times (5 \times 5 \times 4) \times 4 = 100 \times 100 \times 4 = 40{,}000$.

The product is 40,000.

FOLLOW-UP: How many digits are there in the value of the product $2^{17} \times 5^{18}$? [18]

1B

heart

1B *Strategy: Begin by analyzing the two clues that involve the same sticker.*

The puppy is involved in two clues. Number the positions 1 to 5, beginning from the left. Since the puppy is three positions to the left of the kitten, the puppy must be in position 1 or 2, and the kitten in position 4 or 5 respectively. Since the star is one to the left of the puppy, the puppy must be in position 2, the star in position 1 and the kitten in position 5. Only positions 3 and 4 are still available. Since the rainbow is to the right of the heart, the heart is in position 3 and the rainbow in position 4.

The middle position, position 3, has the heart.

FOLLOW-UP: How many arrangements of 5 different stickers are possible? [120]

1C

279

1C *Strategy: Use the process of elimination.*

Since the digits are in increasing order and the difference between the greatest and least is 7, only two patterns are possible: 1*A*8 or 2*B*9. The choice 1*A*8 is not possible since the number must be greater than 200. When a number is divisible by 9, the sum of its digits is also divisible by 9. That means 2 + *B* + 9 must be a multiple of 9 so *B* = 7.

The three-digit number is 279.

It is worth noting that the information given about the number being greater than 200 was unnecessary to solve this problem. If we tried to use the first possible arrangement, 1*A*8, there would be two possible values for *A* since the sum of the digits would have to be a multiple of 9: 1 + 0 + 8 and 1 + 9 + 8. Neither of these would result in a 3-digit number with the digits in increasing order so both of these possibilities would have to be rejected.

1D

76
sq m

1E

2178

1D METHOD 1: *Strategy: Use the additive approach.*

Compute the sum of the areas of the two horizontal sections of the pathway and add it to the sum of the three vertical sections of the pathway, excluding the overlapping portion. The length of the two horizonal sections is 1 + 10 + 1 + 10 + 1 = 23. $2 \times (23 \times 1) + 3 \times (10 \times 1) = 46 + 30$. **The walkway is 76 sq m.**

METHOD 2: *Strategy: Use a subtraction approach.*

Compute the area of the entire outermost rectangular region and then subtract the areas of the two square gardens plots. Since the length of the entire rectangular region is 23 m and the width is 1 + 10 + 1 = 12, the area is $12 \times 23 = 276$ sq m. Each square garden plot is $10 \times 10 = 100$ sq m. The area of the walkway is 276 – (100 + 100) = 76 sq m.

FOLLOW-UP: Given that 5 "neeps" equals 1 meter, find the result to the original problem in "square neeps". [1900 square neeps]

1E *Strategy: Use numerical reasoning.*

Notice that the product contains the initial digits in the reverse order. Since the product contains four digits $M = 1$ or $M = 2$. If $M = 1$, then $4 \times H$ must end in 1, but no multiple of 4 ends in 1. Thus M must be 2. So far, we have:

$$\begin{array}{r} 2ATH \\ \times\ 4 \\ \hline HTA2 \end{array}$$

The only multiples of 4 ending in a 2 are $4 \times 3 = 12$ and $4 \times 8 = 32$. Since H must be greater than M, $H = 8$. Our tableau now is:

$$\begin{array}{r} 2AT8 \\ \times\ 4 \\ \hline 8TA2 \end{array}$$

Since there was no carry in the final product $4 \times M = H$ [$4 \times 2 = 8$], then $A = 1$, yielding:

$$\begin{array}{r} 21T8 \\ \times\ 4 \\ \hline 8T12 \end{array}$$

Since $4 \times 8 = 32$, when we multiply $(4 \times T) + 3$ (including the carry of 3), we obtain a two-digit number ending in 1. $(4 \times 7) + 3 = 31$ is the only possible solution. $T = 7$, and our original product is $2178 \times 4 = 8712$.

$$\begin{array}{r} 2178 \\ \times\ 4 \\ \hline 8712 \end{array}$$

The value of the original 4-digit number represented by *MATH* is 2178.

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Division E
Mathematical Olympiads for Elementary & Middle Schools
December 10, 2013
Contest 2

SOLUTIONS AND ANSWERS

2A 15

2A METHOD 1: *Strategy: Draw a simple diagram.*

Use x's to represent unnamed people. Count the number of x's between them.

Gregor x x x x x x x x x x x x x x x Harold

There are 15 x's or people between Gregor and Harold.

METHOD 2: *Strategy: Use Subtraction.*

Gregor and Harold are two of the 17 people. Subtract these two people from the 17 in the line and there are 15 people remaining.

FOLLOW-UP: A fence consists of 20 poles, each spaced 2 feet apart. How long is the fence? [38 feet]

2B 35

2B METHOD 1: *Strategy: Establish a pattern in the first group of 10's.*

There are five odd 2-digit numbers in the thirties: 31, 33, 35, 37, and 39. There will be five in the forties, five in the fifties, and so on, all the way up to the nineties. Therefore there are 7 groups of 5 numbers each. **There are $7 \times 5 = 35$ two-digit odd numbers greater than 30.**

2C 4

METHOD 2: *Strategy: Realize that half the 2-digit numbers starting with 30 are odd.*

Since there are $99 - 30 = 69$ two-digit numbers greater than 30. Since they start and end with an odd number, 31 and 99, 35 must be odd and 34 must be even.

METHOD 3: *Strategy: Arrange the numbers in a grid for easier counting.*

Create a table using the ten's digits to determine the row and the unit's digits to determine the column.

	1	**3**	**5**	**7**	**9**
3	31	33	35	37	39
4	41	43	45	47	49
5	51	53	55	57	59
6	61	63	65	67	69
7	71	73	75	77	79
8	81	83	85	87	89
9	91	93	95	97	99

2D 9

FOLLOW-UP: How many 3-digit numbers are divisible by 3? [300]

2E 17

2C METHOD 1: *Strategy: The average of a set of values is the sum of the values divided by the number of values in the set.*

If the sum of five numbers divided by 5 is 10, then the sum must be $5 \times 10 = 50$. If the sum of six numbers divided by 6 is 9, then the sum of the six numbers is $6 \times 9 = 54$. **Therefore the number that was added to the original five numbers is $54 - 50 = 4$.**

METHOD 2: *Strategy: Apply algebraic techniques.*

Let N be the number added to the original five numbers and let S be the sum of these numbers. Then $S \div 5 = 10$ so $S = 50$. We also know that $(50 + N) \div 6 = 9$. Multiply each side of this equation by 6 to get $50 + N = 54$. Subtract 50 from each side of the equation to find that $N = 4$.

FOLLOW-UP: Twenty consecutive even integers are selected. David finds the average of the first ten and then finds the average of the last ten integers. He then subtracts the smaller average from the larger average. What is his result? [20]

2D *Strategy: Find the number of ways three-dimensional rectangular solids can be made from a given number of cubes.*

Since each student is given 48 identical cubes to glue together, and all the solids have different dimensions, determine how many ways the product of 3 factors is 48. List the factors of 48: 1, 2, 3, 4, 6, 8, 12, 16, 24, and 48. Start with the largest factor and then find the two smaller factors: $48 \times 1 \times 1$, $24 \times 2 \times 1$, $16 \times 3 \times 1$, $12 \times 4 \times 1$, $12 \times 2 \times 2$, $8 \times 6 \times 1$, $8 \times 3 \times 2$, $6 \times 4 \times 2$, and $4 \times 4 \times 3$. **There are a total of 9 different rectangular solids so there are at most 9 children in the class.**

2E METHOD 1: *Strategy: Create a table of numbers the marker can land on.*

Examine the table and note that these are the numbers that can be gerated using multiples of 3 and multiples of 10. Therefore, the numbers not in the table cannot be generated. These numbers are: 1, 2, 4, 5, 7, 8, 11, 14, and 17. Notice that 18, 19, and 20 are in the table. BY adding 3 to each of these all of th remaining whole numbers can be formed.

	0	3	6	9	12	15	18	…
0	0	3	6	9	12	15	18	…
10	10	13	16	19	22	25	28	…
20	20	23	26	29	32	35	38	…
30	30	33	36	39	42	45	48	…
…	…	…	…	…	…	…	…	…

Therefore the greatest whole number on the number linethat cannot be landed on is 17.

METHOD 2: *Strategy: Consider multiples of 3.*

Every multiple of 3 can be landed on: 3, 6, 9, 12, 15, 18, 21, 24, 27, and 30. The units digit can be every possible digit. By adding 10 to each of these numbers, the units digit remains the same. Therefore the greatest number that is not in this list and cannot be formed by adding 10 is 17.

FOLLOW-UP: Chicken nuggets are sold in orders of 6, 9 and 20. What is the greatest number of nuggets that cannot be purchased exactly? [43]

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Division E — Mathematical Olympiads for Elementary & Middle Schools — January 14, 2014 — Contest 3

SOLUTIONS AND ANSWERS

3A 42

3A METHOD 1: *Strategy: Compose/Decompose or balance the equation.*

Recognize that both sides of the equal sign must have the same value. The 40 on the left side is represented by the 5×8 on the right side. The 6×7 on the left is represented by the *N* on the right side. **Therefore, $N = 6 \times 7 = 42$.**

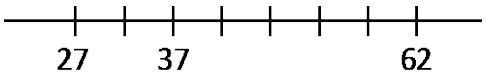
METHOD 2: *Strategy: Multiply and divide.*

Multiply each side of the equation separately. Then $1680 = 40N$. Divide each side by 40 to get $N = 42$.

3B 37 cm

3B *Strategy: Draw a diagram.*

Realize that although there are 8 marks, there are only 7 spaces that take up the distance from the first mark to the last mark. This means that each space between marks is 1/7 of the distance from 27 to 62. First find the distance between the first and last mark and then divide that distance by 7: $(62 - 27) \div 7 = 35/7 = 5$. Count by 5's to find the value at the third mark. **Therefore the value of the third mark is 27 + 5 + 5 = 37.** [Verify that this is the case by determining all 8 marks: 27, 32, 37, 42, 47, 52, 57, and 62.]



3C 80

3C *Strategy: Use the rule for divisibility by 3.*

The divisibility rule for 3 states that if a number is a multiple of 3, then the sum of its digits is also a multiple of 3. Since 541*A*3 is a 5-digit number divisible by 3, and 5 + 4 + 3 = 12 is already divisible by 3, then 1 + *A* must be a multiple of 3. Thus *A* can be 2, since 1 + 2 = 3. It can also be 5 or 8 since 1 + 5 = 6 and 1 + 8 = 9. It follows that *A* can be 2, 5, or 8. Similarly, this is true for *B* and *C*. Since we want the product $A \times B \times C$, it doesn't matter which number is assigned to each letter. **The product $A \times B \times C = 2 \times 5 \times 8 = 80$.**

FOLLOW-UPS: (1) What is the sum of A + B + C if the original numbers were 542A3, 542B3, and 542C3? [12], *(2) Examine the next 2 cases to find a pattern for the sum of A + B + C: case 1 - 543A3, 543B3, 543C3; case 2 - 544A3, 544B3, 544C3.*
[case 1: 4 possible values for the missing digit so no solution can be found; case 2: 15]

3D 29

3E 7501

3D METHOD 1: *Strategy: Divide the sum of the numbers by the number of values added to get the average or middle page number.*

Since the sum of the 11 pages is 374, the average or middle page number is 374/11 = 34. Count backwards 5 pages to get the lowest numbered page: 34, 33, 32, 31, 30, and finally 29. **The first page number in Chapter 3 is 29.** List all 11 page numbers to check the result: 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, and 39.

METHOD 2: *Strategy: Represent the page numbers algebraically.*

Let n be the lowest page number. Then $n + 1$ is the next numbered page. Each page number is one more than the previous page number so the remaining numbered pages are $n + 2$, $n + 3$, $n + 4$, $n + 5$, $n + 6$, $n + 7$, $n + 8$, $n + 9$, and $n + 10$. The sum of these 11 numbers is $11n + 55 = 374$. Subtract 55 from each side of the equation to get $11n = 319$. Divide each side by 11 to get $n = 29$.

3E *Strategy: Consider carrying when adding numbers whose sum is greater than 10.*

The only way to get a 5-digit sum by adding a 4-digit number and a 3-digit number is if carry-overs occur. Therefore, *RARES* begins with 101 and $T = 9$ and $R = 1$. Next, look at the hundreds places ($E + E = 11$) to get that $E = 5$. Next, look at the tens place ($N + N = 15$) to get the $N = 7$. (Note: $T + T = 18$ so $S = 8$). **So, the value of the word *NEAR* is 7501**.

FOLLOW-UP: $AA \times AA = 3BCA$. Each letter represents a different digit. AA is a 2-digit number and 3BCA is a 4-digit number. What is the value of $CC \times CC$? [484]

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Division E — Mathematical Olympiads for Elementary & Middle Schools — February 11, 2014 — Contest 4

SOLUTIONS AND ANSWERS

4A *Strategy: Factor first (decomposition).*

Since the three terms have a common factor of 10, apply the distributive property. **This results in the expression 10(234 + 342 + 423) = 10(999) = 9990**.

FOLLOW-UP: What is the value of $7 \times 49 + 7 \times 21 + 2 \times 30 + 5 \times 30$? [700]

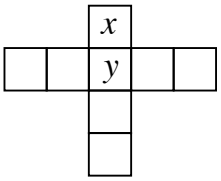
4B **METHOD 1**: *Strategy: Use the sum of all the numbers.*

The sum 1 + 2 + 3 + 4 + 5 + 6 is equal to 21. If we add the numbers in the boxes going across and the numbers in the boxes going down, we obtain 11 + 11 = 22. In doing this, every box is counted once, except for the box labeled X, which is counted twice. It must be filled with the difference of the two results. **The number in the box marked X is 22 – 21 = 1**.

METHOD 2: *Strategy: Find numbers whose sum is 11.*

From the number list given, find 4 numbers whose sum is 11. Since 1 + 2 + 3 + 4 = 10, we need to replace the 4 with a 5. The numbers going across must be 1, 2, 3, and 5. Since 6 and 4 are the only numbers remaining they must go in the vertical column. To make a sum of 11, the number in the box marked X, must be **1**.

FOLLOW-UP: Place the numbers 1, 2, 3, 4, 5, 6, 7, and 8 in the grid shown so that one number is in each box and the sum of the numbers in both the row and the column have a sum of 20. If xy represents the product of the numbers in the boxes labeled x and y, how many different values are possible for xy. [7]



4C *Strategy: Work with factors.*

Consider the integer factors of the three given numbers: $21 = 1 \times 21$ or 3×7, $33 = 1 \times 33$ or 3×11, and $35 = 1 \times 35$ or 5×7. Since the dimensions of the upper left rectangle must be the same as one dimension of the rectangles in the upper right and lower left positions, the dimensions of the upper left rectangle must be 3 and 7. Therefore the dimensions of the upper right rectangle are 3 and 11, and the lower left must be 5×7. **The big rectangle has dimensions 3 + 5 = 8 by 7 + 11 = 18 and the perimeter is 2(8) + 2(18) = 52**.

4A	4B	4C	4D	4E
9990	1	52	10	72

4D *Strategy: Sum the digits to discover if a number is divisible by 9.*

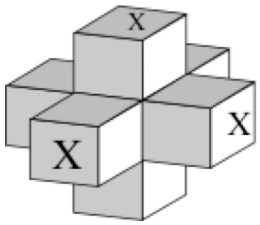
If the sum of the digits is a multiple of 9 then so is the original number. Therefore when $A + 5 + B = 9$ or 18, the 3-digit number $A5B$ will be divisible by 9. If $A + 5 + B = 9$ then $A + B = 4$. There are four possible numbers that satisfy this condition: 153, 252, 351, and 450. When $A + 5 + B = 18$, $A + B = 13$ and there are 6 additional solutions: 954, 855, 756, 657, 558 and 459. **There are many three-digit numbers that are divisible by 9, but only 10 three-digit numbers of the form A5B.**

FOLLOW-UP: How many different three-digit numbers of the form A5B are divisible by 11? Note that A and B could be the same digit. [8]

4E **METHOD 1**: *Strategy: Use symmetry to help find the answer.*

The surface area of the front face is 8. The surface area of the space left when the middle cube is removed is 4. Therefore the total surface area of the front face unit is 8 + 4 = 12. **Since there are 6 similar faces the total surface area of the block is $6 \times 12 = 72$.**

METHOD 2: *Strategy: Work with the missing space.*



If the original cube was left alone its surface area would be $6(3 \times 3) = 54$. Consider the surface area of the shape that is removed. This 30-sided figure has 24 faces that are new exposures for the cubes surface and 6 faces, 3 visible and marked with an X, that once were parts of faces of the cube. Therefore the surface area is 54 + 24 – 6 = 72.

METHOD 3: *Strategy: Consider each kind of "surface".*

To calculate the number of 1×1 squares on the outer surface of the modified cube, recognize that there are 6 faces each having 8 exposed squares. Therefore there are $6 \times 8 = 48$ square units on the outer surface. To calculate the number of exposed squares inside the modified cube, realize that the center square removed from each face leaves 4 exposed squares. Since there are 6 faces with the center cube removed, there will be $6 \times 4 = 24$ exposed interior squares. The total number of exposed squares is 48 + 24 = 72, the surface area of the modified cube.

FOLLOW-UPS: (1) What is the surface area when sixty-four $1 \times 1 \times 1$ cubes are placed in a row. [258] *(2) What is the surface area when sixty-four $1 \times 1 \times 1$ cubes are arranged to form a square.* [160] *(3) What is the surface area when sixty-four $1 \times 1 \times 1$ cubes are arranged to form a cube.* [96]

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Division E — Mathematical Olympiads for Elementary & Middle Schools — March 11, 2014 — Contest 5

SOLUTIONS AND ANSWERS

5A

5A *Strategy: To maximize one number, minimize another.*

Since we want the greatest number of pencils in the last box place the smallest possible numbers in the first five boxes. This results in boxes containing 1, 2, 3, 4, and 5 pencils for a total of 15. **Since Julio has 23 pencils, 23 – 15 = 8 pencils can be placed in the sixth box**.

8

5B

5B METHOD 1: *Strategy: Eliminate the fractions.*

Subtract $\frac{1}{6}$ from both sides of the equation to result in $\frac{1}{N}=\frac{1}{5}-\frac{1}{6}=\frac{6}{30}-\frac{5}{30}=\frac{1}{30}$. Now invert both fractions. **The value of *N* that satisfies the equation is 30**.

30

METHOD 2: *Strategy: Eliminate the fractions.*

In any equation we can eliminate the denominators of fractions by multiplying by a common denominator. The LCD is 30*N*. If we multiply each term by this expression we get 6*N* = 5*N* + 30 so *N* = 30.

5C

5C METHOD 1: *Strategy: Draw some lines.*

Draw XB to divide the shaded region into two triangles. Then draw lines from X perpendicular to sides AB and BC. Since the area of the given square is 64 each side is 8 and BE = 6 while BF = 2. The length of the perpendicular segments (altitudes of ΔXEB and ΔXBF) is 4. **The area of the shaded region is the sum of the areas of ΔXEB and ΔXBF = (0.5)(4)(6) + (0.5)(4)(2) = 16**.

16

5D

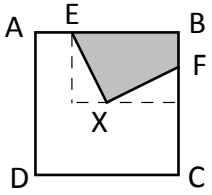
METHOD 2: *Strategy: Draw different lines.*

Draw AX and BX. Notice that ΔAXE and ΔBXF are the same size and shape (congruent) so they have the same area. The area of the shaded region will be the same as the area of ΔAXB which is 1/4 the area of the original square or (1/4)(64) = 16.

50

METHOD 3: *Strategy: Subtract areas.*

Create a rectangle around the shaded region with a length of 6 and a width of 4. The area of this rectangle is 6 × 4 = 24. Find the area of the two triangles in the rectangle but outside the shaded region. Both triangles are right triangles and have the same area. Each area is (0.5)(4)(2) = 4. Therefore the area of the shaded quadrilateral is 24 – (4 + 4) = 16.



5E

5

FOLLOW-UP: Square ABCD has point X at its center. E is one-quarter of the way from A to B and F is one-third the way from B to C. Find the area of quadrilateral EBFX. [17 1/3]

5D METHOD 1: *Strategy: Find the common multiples of 2, 3, and 4.*

The number of pieces that person A has must be divisible by 2, 3, and 4. The least common multiple of those numbers is 12. But, person D would have 1/4 of 12, which would be fewer than 4 pieces. The next larger common multiple is 24; so person A would have 24, Person B would have 12, Person C would have 8 and Person D would have 6.
Therefore there are 24 + 12 + 8 + 6 = 50 pieces.

METHOD 2: *Strategy: Work backwards from D's minimum 4 pieces.*

If D has 4 then A has 16, B has 8, and C would not have a whole number of pieces.
If D has 5 then A has 20, B has 10, and C would not have a whole number of pieces.
If D has 6 then A has 24, B has 12, and C has 8. Together they have 50 pieces.

FOLLOW-UP: Maria arranges her checkers in piles. If she puts them in piles of 6, there will be none left over. If she puts them in piles of 8 there will be none left over. If she puts them in piles of 9 there will be none left over. Maria has fewer than 100 checkers. If she puts them in piles of 10, how many will be left over? [2]

5E METHOD 1: *Strategy: Apply the concept of average.*

For the average to fall to 96 when Person X joins the group, each of the current members must give 4 of their own marbles to Person X. Since he started with 80, he needs to receive 16 marbles to arrive at the average number of 96. Therefore, he must receive a total of 16 marbles. **Since he receives 4 from each child there must be 4 other children or 5 counting Person X**.

METHOD 2: *Strategy: Construct a table.*

Construct a table that keeps track of how many people there could be:

Number of People (without X)	2	3	4
Total number of marbles	200	300	400
Add 80 marbles from person X	280	380	480
Average with person X	280/3 = 93.3	380/4 = 95	480/5 = 96

Notice that the only possibility is 4 people plus person X or a total of 5 people.

FOLLOW-UPS: There are 10 students in Ms. Pemberton's class. On the day of a test, Joanne, a student in the class was absent. The average of the remaining students was 90. What grade on the make-up exam must Joanne receive to raise the class average to 91? [100]

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