Division **E**

Mathematical Olympiads

November 13, 2012

for Elementary & Middle Schools

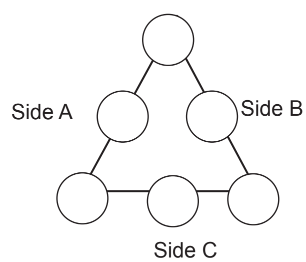
Contest **1**

1A ***Time: 3 minutes***
Find the value of 87 – 76 + 65 – 54 + 43 – 32 + 21 – 10.

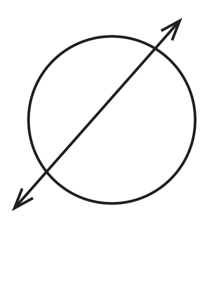
1B ***Time: 5 minutes***
Grace chooses five different numbers from the list 1, 2, 3, 4, 5, 6, 7, 8, 9,10. Two of those numbers are 4 and 5, and they are the only two numbers she picks that differ by 1. What is the greatest possible sum of the five numbers?

1C ***Time: 5 minutes***
A 7×7 square is marked off into forty-nine 1×1 small squares. Each of the small squares along the edges of the large square is painted blue. How many small squares are painted blue?

1D ***Time: 6 minutes***
The numbers 1, 2, 3, 4, 5, and 6 are placed in the diagram, one in each circle. The sum of the numbers along Side A is 13, along Side B is 13, and along Side C is 6. What number is in the circle at the top of the diagram?



1E ***Time: 6 minutes***
The region inside the circle can be cut into 2 parts by drawing 1 line through it, as shown. What is the greatest number of parts that can be formed by drawing 4 lines through the region?



Please fold over on line. Write answers on back.

Division

E

Mathematical Olympiads

November 13, 2012

for Elementary & Middle Schools

Contest

1

1A *Student Name and Answer*

1B *Student Name and Answer*

1C *Student Name and Answer*

1D *Student Name and Answer*

1E *Student Name and Answer*

Please fold over on line. Write answers in these boxes.

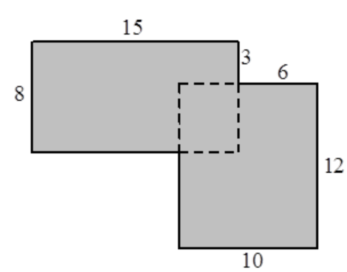
2A *Time: 3 minutes*
What is the value of (47 × 8) + (8 × 27) + (26 × 8)?

2B *Time: 5 minutes*
Juan uses the digits 1, 2, 3, 4, 5, and 6 to make two 3-digit numbers. Each digit is used once. The numbers are subtracted. What is the greatest possible difference?

2C *Time: 5 minutes*
Four volunteers can pack 12 boxes every 30 minutes. How many ***additional*** volunteers are needed to pack 72 boxes every hour? [Assume all volunteers work at the same rate.]

2D *Time: 6 minutes*
How many 3-digit numbers are multiples of 21?

2E *Time: 7 minutes*
The region is formed by two overlapping rectangles. Determine the area of the entire region. The lengths shown are in cm.



Please fold over on line. Write answers on back.

Division **E**

Mathematical Olympiads

December 11, 2012

for Elementary & Middle Schools

Contest **2**

2A *Student Name and Answer*

2B *Student Name and Answer*

2C *Student Name and Answer*

additional volunteers

2D *Student Name and Answer*

2E *Student Name and Answer*

sq cm

Please fold over on line. Write answers in these boxes.

Division **E** | Mathematical Olympiads | **January 8, 2013** | for Elementary & Middle Schools | Contest **3**

3A *Time: 3 minutes*

What number does ◊ represent if

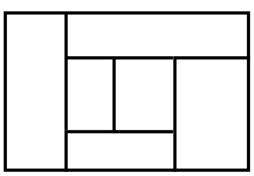
⌂ + ◊ = 20 and ⌂ + ⌂ = 24?

3B *Time: 4 minutes*

How many numbers between 10 and 99 have digits that differ by 3?

3C *Time: 5 minutes*

How many rectangles are in this picture?



3D *Time: 6 minutes*

A teacher surveyed 24 students and discovered that

- 18 of them like to play video games;
- 15 of them like to go to the movies, and
- 2 of them do not like either playing video games or going to the movies.

How many of the 24 students like both activities?

3E *Time: 7 minutes*

Every digit from 1 through 9 appears in the addition problem on the right. Each digit is used exactly once. The addition problem yields a correct answer. What three-digit number is the answer to the addition problem?

$$\begin{array}{r} 5\ 6\ 7 \\ +\ \square\ \square\ \square \\ \hline \square\ \square\ \square \end{array}$$

Please fold over on line. Write answers on back.

Division

E

Mathematical Olympiads

January 8, 2013

for Elementary & Middle Schools

Contest

3

3A *Student Name and Answer*

3B *Student Name and Answer*

3C *Student Name and Answer*

3D *Student Name and Answer*

3E *Student Name and Answer*

Please fold over on line. Write answers in these boxes.

Division **E**

Mathematical Olympiads

February 12, 2013

for Elementary & Middle Schools

Contest **4**

4A ***Time: 4 minutes***

Chloe divides the number *N* by 14 correctly and gets 5. Mia uses the same value of *N* as Chloe but she misreads the division sign as an addition sign and finds the sum. What answer does Mia get?

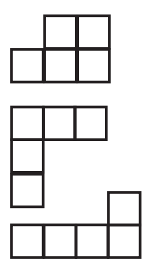
4B ***Time: 4 minutes***

Sara starts with the number 9 and counts by 7s. This results in the sequence 9, 16, 23, 30, 37, ….
What is the twenty-fourth number in Sara's sequence?

4C ***Time: 5 minutes***

What is the least prime number that is the sum of 3 different prime numbers?

4D ***Time: 6 minutes***



Jen plays a game where three kinds of pieces, each with an area of 5 sq units, must be placed (without overlap) on a checkerboard. The kinds of pieces are shown. They may be rotated.The checkerboard is 8 units high by 8 units wide. What is the greatest number of pieces that Jen can place on the checkerboard?

4E ***Time: 7 minutes***

Lara starts at the bottom of a long staircase. She climbs exactly $\frac{2}{3}$ of the stairs. Then, she goes back down exactly $\frac{1}{2}$ of the way to the bottom. From that spot, she climbs exactly $\frac{2}{3}$ of the way to the top. Finally, from there, she climbs 6 stairs to reach the top. How many stairs are in the staircase?

Please fold over on line. Write answers on back.

Division

E

Mathematical Olympiads

February 12, 2013

for Elementary & Middle Schools

Contest

4

4A *Student Name and Answer*

4B *Student Name and Answer*

4C *Student Name and Answer*

4D *Student Name and Answer*

4E *Student Name and Answer*

Please fold over on line. Write answers in these boxes.

Division E | Mathematical Olympiads for Elementary & Middle Schools | March 12, 2013 | Contest 5

5A ***Time: 4 minutes***
What number is one-half of one-third of one-fourth of 240?

5B ***Time: 5 minutes***
Beginning with the box labeled 1, form a path by placing the numbers 2 through 16 in the boxes, so that each consecutive number occupies an adjacent box (no diagonal path allowed). Some numbers are already written in. Find the sum of the numbers in the starred (*) boxes.

3			
	1		
5	*		*
		16	

5C ***Time: 5 minutes***
Group A has 10 numbers with an average of 10. Group B has 20 numbers with an average of 20. Group C has 30 numbers with an average of 30. Group D has 40 numbers with an average of 40. The four groups are put together. What is the average of the combined group?

5D ***Time: 5 minutes***
Judy has two more sisters than she has brothers. Her brother, Mark, has twice as many sisters as he has brothers. How many children are in their family?

5E ***Time: 7 minutes***
A 4×4×4 cardboard box is going to be filled with colored 1×1×1 blocks. One red block is placed into the corner of the box. Next, the least number of orange blocks are used to completely hide the red block from view. Then, the least number of yellow blocks are used to hide the orange blocks from view. At this point, how many blocks have been placed into the box?

Please fold over on line. Write answers on back.

Division **E**

Mathematical Olympiads

March 12, 2013

for Elementary & Middle Schools

Contest **5**

5A *Student Name and Answer*

5B *Student Name and Answer*

5C *Student Name and Answer*

5D *Student Name and Answer*

5E *Student Name and Answer*

Please fold over on line. Write answers in these boxes.

Division **E** | Mathematical Olympiads | **November 13, 2012** | for Elementary & Middle Schools | Contest **1**

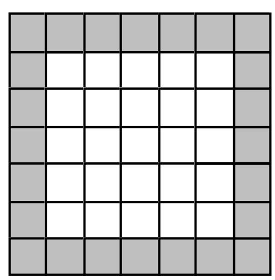
SOLUTIONS AND ANSWERS

1A *Strategy: Use the pattern in the numbers to simplify the arithmetic.*
Group the numbers in pairs: **87 – 76 + 65 – 54 + 43 – 32 + 21 – 10** =
(87 – 76) + (65 – 54) + (43 – 32) + (21 – 10) =
11 + 11 + 11 + 11 **= 44**

1B **METHOD 1:** *Strategy: Start with the greatest number.*
To get the greatest sum, start by choosing 10. Then 9 can't be used so choose 8. 7 can't be used, and neither can 6 because it's adjacent to 5 which is given as one of the numbers. 4 and 5 must be included, so 3 can't be used. Choose 2 as the final number. **The greatest possible sum of Grace's numbers is** 10 + 8 + 5 + 4 + 2 = **29**.

METHOD 2: *Strategy: Start with the known numbers.*
Two of the numbers are 4 and 5. Neither 3 nor 6 can be used because they are adjacent to the two numbers already used. The other three numbers must be chosen from 1,2,7,8,9, and 10. Only two numbers can be chosen from 7, 8, 9, and 10, and therefore either 1 or 2 must be used. The greatest possible sum of Grace's numbers is 10 + 8 + 5 + 4 + 2 = 29.

1C **METHOD 1:** *Strategy: Draw a picture.*
Draw the 7×7 square. Shade the small squares on the edges and count them. **24 of the small squares are painted blue.**



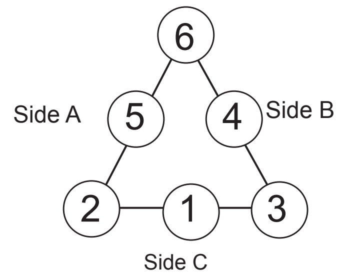
METHOD 2: *Strategy: Count the small squares along one edge.*
7 small squares along each of the 4 sides of the large square give a total of 28. But each of the 4 corner squares has been counted twice. Thus there are a total of 28 – 4 = 24 squares that are painted blue.

METHOD 3: *Strategy: Count the squares that are not painted.*
The unpainted squares form a 5×5 array of 25 squares inside the large square. Then 49 – 25 = 24 squares are painted blue.

> *Follow-Ups: (1) A 10×6 rectangle is marked off into 60 small squares. Each of the squares along the edges of the rectangle and each of the squares adjacent to those border squares is painted red. How many of the small squares are painted red?* [48] *(2) A rectangular photo that is 16 cm by 20 cm is surrounded by a frame that is 2 cm in width. What is the area of the frame?* [160 sq cm]

1A	44
1B	29
1C	24
1D	6
1E	11

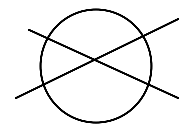
1D *Strategy: Find all sets of 3 numbers with the given sums.*
The only sets of 3 different numbers chosen from the given values that have a sum of 13 are {2,5,6} and {3,4,6}. These are the numbers along sides A and B in the diagram. **The circle at the top** is on both sides and therefore **contains 6**.
To check, note that the only 3 numbers that have a sum of 6 are 1, 2, and 3. These must be along the bottom. This leads to the solution to the right.



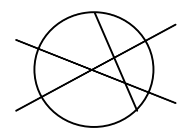
(Sides A and B may be interchanged.)

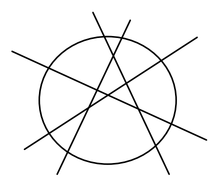
1E *Strategy: Draw the picture and count carefully.*
In order to have the maximum number of regions, every new line must be added in such a way that each pair of lines intersect at a unique point in the interiior of the circle

If 2 straight lines are drawn and they intersect, 4 parts are created.



If 3 intersecting lines are drawn, at most, 7 parts may be created.





If four lines are drawn, the greatest number of parts we can create is 11.

FOLLOW-UPS: (1) What is the greatest number of intersection points of 4 lines in the plane? [6] *(2) Can you find a pattern in the number of regions inside the circle formed by 1, 2, 3, or 4 lines? How many regions inside the circle can be formed by 5 lines? [16] By 6 lines?* [22]

NOTE: Other FOLLOW-UP problems related to some of the above can be found in our two contest problem books and in "Creative Problem Solving in School Mathematics." Visit www.moems.org for details and to order.

SOLUTIONS AND ANSWERS

2A	800
2B	531
2C	8
2D	43
2E	220

2A **METHOD 1:** *Strategy: Use the distributive property of multiplication over addition.*
(47 × 8) + (8 × 27) + (26 × 8) = (47 + 27 + 26) × 8 = 100 × 8 = **800.**

METHOD 2: *Strategy: Perform the operations as indicated.*
(47 × 8) + (8 × 27) + (26 × 8) = 376 + 216 + 208 = 800.

2B *Strategy: Maximize the minuend; minimize the subtrahend.*
Use the three greatest digits in the minuend and arrange them from greatest to least. Arrange the other three digits from least to greatest in the subtrahend. 654 – 123 = **531 is the greatest possible difference.**

Follow-Up: The ten digits are each used once to form two 5-digit even numbers whose difference is a maximum. What are the numbers? [98,756 – 10,234 = 88,522]

2C **METHOD 1:** *Strategy: Find how much 4 volunteers can do in 1 hour.*
If 4 volunteers can pack 12 boxes every 30 minutes, then 4 volunteers can pack 24 boxes every hour (60 minutes). In order to pack 72 boxes in an hour, since 72 ÷ 24 = 3, three times as many volunteers are needed. So 12 volunteers are needed to do the job; therefore, **8 additional volunteers are needed.**

METHOD 2: *Strategy: Find how many boxes 1 volunteer can pack in an hour.*
If 4 volunteers can pack 12 boxes every 30 minutes, then 1 volunteer can pack 1/4 as many in 30 minutes, or 3 boxes. One volunteer can then pack 6 boxes in an hour. To pack 72 boxes requires 12 volunteers, and so 8 additional volunteers are needed.

Follow-Up: If 8 volunteers can pack 12 boxes in 30 minutes, how long would it take 6 volunteers to pack 24 boxes? [80 minutes]

2D **METHOD 1:** *Strategy: Find the least and greatest 3-digit multiples of 21.*
100 ÷ 21 = 4 R16, so 4 × 21 has 2 digits and 5 × 21 = 105 is the least 3-digit multiple of 21; 999 ÷ 21 = 47 R12, so 47 × 21 = 987 is the greatest 3-digit multiple of 21. There are 47 multiples of 21 that are less than 1000, but 4 of them have only 2 digits, so **there are** 47 – 4 = **43 three-digit numbers that are multiples of 21.**

2D **METHOD 2:** *<u>Strategy</u>: List the multiples of 21.*

List all the multiples of 21 until all 3-digit multiples are discovered.

The multiples of 21 that have 3 digits are:

21, 42, 63, 84, 105, 126, 147, 168, 189, 210, 231, 252, 273, 294
315, 336, 357, 378, 399, 420, 441, 462, 483, 504
525, 546, 567, 588, 609, 630, 651, 672, 693, 714
735, 756, 777, 798, 819, 840, 861, 882, 903, 924
945, 966, 987, ***1008.***

In all there are 43 multiples of 21 that have 3 digits.

> ***Follow-Up:*** *How many 3-digit odd numbers are not multiples of 5?* [360; any 3-digit number which ends with 1, 3, 7 or 9]

2E **METHOD 1:** *<u>Strategy</u>: Determine the dimensions of the overlapping region.*

Call the large rectangles ABLH and KDEF as shown.

The area of rectangle ABLH is 15 × 8 = 120 sq.cm.

The area of rectangle KDEF is 12 × 10 = 120 sq.cm.

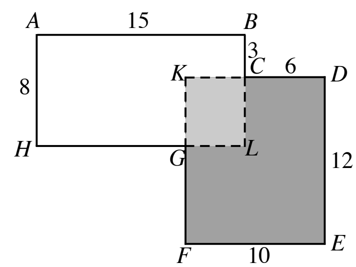
The sum of the areas is 240 sq cm, but this counts the area of rectangle KCLG twice. The area of the entire region is then 240 – the area of KCLG.

KC + 6 = EF = 10, so KC = 4 cm.

LC + 3 = 8, so LC = 5 cm.

The area of rectangle KCGL is 4 × 5 = 20 sq cm,

so **the area of the entire region is** 240 – 20 = **220 sq cm.**

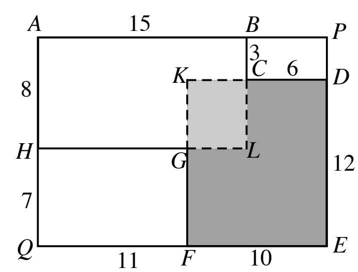


METHOD 2: *<u>Strategy</u>: Determine the area of the surrounding rectangle; subtract unwanted regions.*

Draw rectangle APEQ. PE = 12 + 3 = 15, so HQ = 7.

AP = 15 + 6 = 21, so QF = 11. The area of the original region = area of rectangle APEQ – (area of HGFQ + area of BPDC) = 15 × 21 – (7 × 11 + 3 × 6) = 315 – (77 + 18) = 315 – 95 = 220 sq cm.

> ***Follow-Up:*** *What is the perimeter of the shaded region?* [72 cm]



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Division **E**

Mathematical Olympiads

January 8, 2013

for Elementary & Middle Schools

Contest **3**

SOLUTIONS AND ANSWERS

3A *Strategy: Solve for ⌂ first.*

⌂ + ⌂ = 24 Therefore, ⌂ = 12.

⌂ + ◊ = 20 or 12 + ◊ = 20.

Therefore, ◊ **represents 8**.

FOLLOW-UP: What whole numbers have a product of 144 and a sum of 25? [16 and 9]

3B *Strategy: Make an organized table.*

For each possible tens digit, 1 through 9, determine each possible units digit (that differs by 3 from the tens digit). Keep in mind that it could differ either above or below.

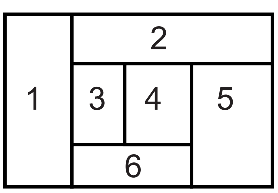
Differs Above	14	25	36	47	58	69			
Tens Digit	1	2	3	4	5	6	7	8	9
Differs Below			30	41	52	63	74	85	96

Make sure that there are not any repeats, then count up the numbers in the list. **There are 13 numbers between 10 and 99 whose digits differ by 3.**

FOLLOW-UPS: (1) How many 2-digit numbers have a tens digit that is less than the ones digit? [36] *(2) How many 3-digit numbers have a tens digit that is less than the ones digit and a hundreds digit that is less than the tens digit?* [84]

3C *Strategy: Select labels to organize the counting of rectangles.*

Number each individual rectangle (that is, the rectangles that do not have smaller areas inside of them) – see diagram. The selection of numerical labels means that we can keep everything in numerical order.



For example, we will never call a composite rectangle 43 because the digits are not in numerical order; instead, we would call it 34 – this will help prevent counting the same rectangle twice. Now, starting with 1, and only counting upwards, determine all possible rectangles:

1, 123456, 2, 23456, 3, 34, 346, 3456, 4, 5, 6.

There are 11 rectangles in this diagram.

3A	**8**
3B	**13**
3C	**11**
3D	**11**
3E	**891**

3D **METHOD 1:** *Strategy: Draw a diagram.*

Draw 24 boxes to represent the students. Put N in 2 boxes to represent those who like neither. Then put ***V*** in the next 18 boxes to represent those who like video games.

N	N	V	V	V	V	V	V
V	V	V	V	V	V	V	V
V	V	V	V				

Now, starting at the end of the list of boxes, put ***M*** in 15 boxes to represent those who like to go to the movies.

N	N	V	V	V	V	V	V
V	V M	V M	V M	V M	V M	V M	V M
V M	V M	V M	V M	M	M	M	M

Count the boxes with both V and M in them. **11 students like both**.

METHOD 2: *Strategy: Use reasoning.*

Ignore the 2 students who liked neither. The remaining 22 students liked at least 1 of the activities. If the number who liked video games and the number who liked movies are added, the total, 33, is greater than the 22 students who liked one or more of them. Some of the students must have been counted twice. These are the students who liked both. 33 – 22 = 11 students like both.

> ***Follow Up:*** *Suppose in the given problem students are also asked whether they like to play basketball. One liked none of the 3, one liked movies and basketball but not video games, four liked movies and video games but not basketball and two liked video games only. How many liked basketball?* [14]

3E *Strategy: Consider possibilities.*

First, note that both the 8 and the 9 must be a part of the sum, with one of these as the hundreds digit. (We might think to try the only place an 8 or a 9 could go in the missing addend - and 8 in the tens place - to get 567+38__ = 94__. But this does not continue to work.) Next, we must consider how the 1 is used.

Case 1: 567 + __ __1 = 9__8 (but this always fails in the ten's place).

Case 2: 567 + __1__ = 98__ (but the unit's digit carrying is impossible here, as only the numbers 2, 3 and 4 are available to use in the blanks).

Case 3: 567 + __4__ = __1__ (but the unit's digit carrying is impossible here too, as only the numbers 2, 3 and 4 are available to use in the blanks and using the 9 in the ones place yields a 6 in the sum, but 6 was already used.).

Case 4: 567 + __ __4 = __ __1.

Because of the units digit carrying, we must get that **567 + 324 = 891**.

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Division E
Mathematical Olympiads
February 12, 2013
for Elementary & Middle Schools
Contest 4

SOLUTIONS AND ANSWERS

4A *Strategy: Work backwards.*
Before Chloe divides the number N, it is 14 times as large as her final answer, 5. So, we can determine N by calculating $14 \times 5 = 70$. Then, Mia accidentally adds the 14 to the 70 to get her answer: **84**.

4A: 84

4B METHOD 1: *Strategy: Make a chart and look for a pattern.*

	1st	2nd	3rd	4th	5th	…	24th
Sara's Sequence	9	16	23	30	37	…	?
Counting by 7s	7	14	21	28	35	…	168

$24 \times 7 = 168$. Notice that each number in Sara's sequence is 2 greater than the corresponding multiple of 7. Therefore the 24th number in her sequence is 2 greater than 168. **The twenty-fourth number in Sara's sequence is 170.**

METHOD 2: *Strategy: Continue to add 7s.*
If you list your numbers in groups of 10 it will be easier to notice patterns and spot addition errors.

9 16 23 30 37 44 51 58 65 72
79 86 93 100 107 114 121 128 135 142
149 156 163 170

The twenty-fourth number in Sara's sequence is 170.

4B: 170

4C *Strategy: List prime numbers starting with the least.*
2 3 5 7 11 13 17 19 23 29 31 …
Note that 2 is the only even prime. If 2 is among the three primes added, the sum is even and therefore not prime. Eliminate 2 from the list. The least sum is $3 + 5 + 7 = 15$, which is not prime. The next smallest sum of primes is $3 + 5 + 11 = 19$. Therefore, **19 is the least prime number that is the sum of 3 different prime numbers.**

Follow-Up: *What is the least prime number that can be written as the sum of 3 different nonzero perfect square numbers?* [29]

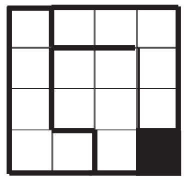
4C: 19

4D: 12

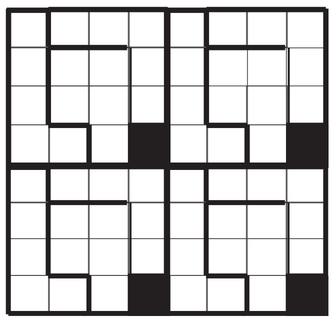
4E: 27

4D *Strategy: First find the greatest possible number that can fit.*

The area of the screen is 8 × 8 = 64 sq units and each piece has an area of 5 sq units. 64 ÷ 5 = 12 R 4, so no more than 12 pieces can be placed on the screen. To see whether 12 pieces can actually fit, place them as compactly as possible. For example, use one of each kind of piece to cover 15 of the 16 squares in a 4 × 4 screen.



The 8 × 8 screen can be divided into four 4 × 4 regions and this arrangement can be placed in each of them as shown below.



The greatest number of pieces Jen can place on the screen is 12.

4E METHOD 1: *Strategy: Draw a diagram to show each action Lara takes.*

Lara climbs exactly $\frac{2}{3}$ of the steps.

Then she goes back down exactly $\frac{1}{2}$ of the steps she just climbed. $\frac{1}{2}$ of $\frac{2}{3} = \frac{1}{3}$.

From that spot, she climbs exactly $\frac{2}{3}$ of the steps above her $\frac{2}{3}$ of $\frac{2}{3} = \frac{4}{9}$.

From there, she climbs 6 stairs to reach the top. Those 6 steps equal $\frac{2}{9}$ of the staircase.

Therefore, $\frac{1}{9}$ of the staircase = 3 steps and $\frac{9}{9}$ (the whole staircase) = 27 steps.
There are 27 stairs in the staircase.

METHOD 2: *Strategy: Guess and check; make a chart.*

The number of steps has to be divisible by 3 and 9, and greater than 9.

Guess	After 1st Climb	After 1st Return (1/2 way)	Steps Remaining	2/3 of Remaining Steps	6 More Steps
18	Step 12	Step 6	12	Step 14	20 (Too Many)
27	Step 18	Step 9	18	Step 21	Step 27

Division **E**

Mathematical Olympiads
March 12, 2013
for Elementary & Middle Schools

Contest **5**

SOLUTIONS AND ANSWERS

5A METHOD 1: *Strategy: Multiply all of the fractions together first.*
(1/2) × (1/3) × (1/4) = 1/24, and (1/24) × 240 = **10**

METHOD 2: *Strategy: Perform the multiplications one at a time.*
(1/2) × (1/3) × (1/4) × 240 = (1/2) × (1/3) × [(1/4) × 240] = (1/2) × (1/3) × 60 = (1/2) × [(1/3) × 60] = (1/2) × 20 = 10

FOLLOW-UP: $900 is divided in half. Those halves are divided into three equal piles, and each of those equal piles is divided equally among five people. How much money will each person receive? [$30]

5B *Strategy: Start where there is only one possible choice.*
The square between 3 and 5 must contain 4. There is then only one possible location for 2 that lies adjacent to 3.

3	**2**		
4	1		
5	*		*
		16	

It seems there are two possible locations for 6. If 6 goes to the right of 5, however, there is no path from that location to accomplish the task (assuring that each consecutive number occupies an adjacent box). So 6 goes in the lower left corner. The other squares may now be filled in.

3	2	**11**	**12**
4	1	**10**	**13**
5	**8**	**9**	**14**
6	**7**	16	**15**

The sum of the numbers in the starred boxes is 8 + 14 = **22.**

5C *Strategy: Use the definition of average.*
The numbers in Group A have a sum of 100 (10 × 10). The numbers in Group B have a sum of 400 (20 × 20). The numbers in Group C have a sum of 900 (30 × 30). The numbers in Group D have a sum of 1600 (40 × 40). Therefore, the sum of all 100 numbers is 100 + 400 + 900 + 1600 = 3000.
The average of the combined group is 3000/100 = **30**.

FOLLOW-UP: Jimmy bikes to a park, a distance of 30 miles, at 10 miles per hour. He bikes home along the same path at 6 miles per hour. What is Jimmy's average speed for the entire trip? [7.5 miles per hour]

5A	10
5B	22
5C	30
5D	13
5E	10

5D *<u>Strategy</u>: Organize the data in two charts.*
Consider the number of brothers and sisters each may have and determine the total number of children in the family. Remember also that Mark has one more sister than Judy, and Judy has one more brother than Mark has.

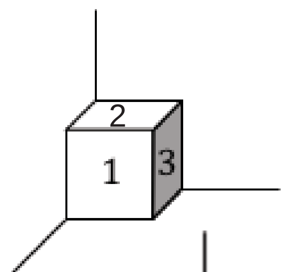
# of brothers that Judy has	0	1	2	3	4	5	…
# of sisters that Judy has	2	3	4	5	6	7	…
# of children including Judy	3	5	7	9	11	13	…

# of brothers that Mark has	1	2	3	4	5	6	…
# of sisters that Mark has	2	4	6	8	10	12	…
# of children including Mark	4	7	10	13	16	19	…

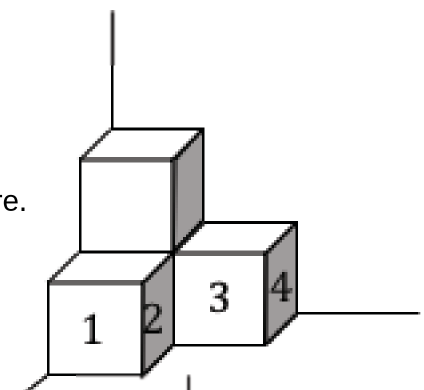
The bolded columns seem to show that both 7 and 13 are possible totals, but only in the circled columns does Mark have one more sister than Judy has and Judy one more brother than Mark has. **There are 13 children in their family.**

5E *<u>Strategy</u>: Place the least number of blocks of each color.*
The red block is in a corner, so 3 faces are visible.

Each of those will require one orange block to hide it, so now there are 4 blocks, one on top of three.

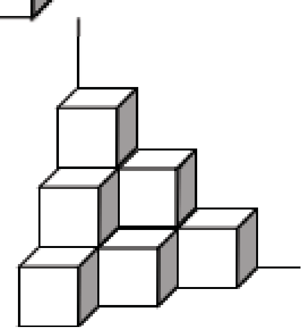


On the lower tier, there are four orange faces visible, but only three yellow blocks are necessary to cover them, for a single yellow block can cover faces 2 and 3 in the picture.



On the second tier, two yellow blocks will cover the tops of the lower orange blocks as well as the sides of the top orange block. Then, one yellow block will cover the top of the uppermost orange block. A total of 6 yellow blocks are needed.

At this point, there are 1 + 3 + 6 = **10 blocks in the box.**



NOTE: *Other* *FOLLOW-UP* *problems related to some of the above can be found in our two contest problem books and in "Creative Problem Solving in School Mathematics."*
Visit <u>www.moems.org</u> for details and to order.