Division **E**

Mathematical Olympiads

NOVEMBER 15, 2011

for Elementary and Middle Schools

Contest **1**

1A *Time: 3 minutes*

Write the value of this expression as a whole number.

(20 + 40 + 60 + 80 + 100+ 120) – (10 + 30 + 50 + 70 + 90 + 110)

1B *Time: 5 minutes*

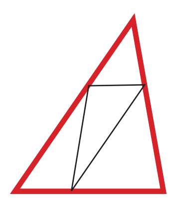
In a group of dogs and their owners, there are exactly 20 heads and 64 legs. How many dogs are in the group?

1C *Time: 5 minutes*

Three friends play a series of 8 games. The winner of each game scores 8 points. Second place scores 3 points. Last place scores 0 points. At the end of 8 games, Keri's score is 20 points. In how many of the games did Keri finish last?

1D *Time: 6 minutes*

How many four-sided figures can be traced, using the lines in this picture?



1E *Time: 7 minutes*

Different letters represent different digits.
If ADD + ADD + ADD = SUMS and A is even, what is the 4-digit number SUMS?

Please fold over on line. Write answers on back.

Division **E**

Mathematical Olympiads

DECEMBER 13, 2011

MATH OLYMPIAD

for Elementary and Middle Schools

Contest **2**

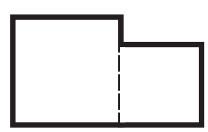
2A ***Time: 3 minutes***
The sum of the digits of the number 2010 is 3. What is the next larger number whose digit-sum is also 3?

2B ***Time: 4 minutes***
In the pattern below, what will be the 78th letter written?

A B B C C D A B B C C D … (*and so on*)

2C ***Time: 5 minutes***
If you triple Jen's age and subtract 16, the result will be the same as when you double her age and add 8. How old is Jen?

2D ***Time: 6 minutes***
This figure consists of two squares. Each side-length is a whole number of centimeters. The combined areas of the squares is 100 sq cm. What is the perimeter of the entire figure?



2E ***Time: 7 minutes***
A pile of coins sits on a table. Sara takes half of the coins plus 4 more. Then Nick takes 2. Then Joe takes 2 more than half of what is left. Finally Selena takes 5. Four coins remain on the table. How many coins were on the table **to start with**?

Please fold over on line. Write answers on back.

Division E

Mathematical Olympiads

JANUARY 10, 2012

for Elementary and Middle Schools

Contest 3

3A *Time: 3 minutes*

How many 2-digit numbers have one digit that is twice the other?

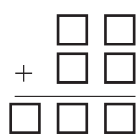
3B *Time: 4 minutes*

In a magic square, the sum of the numbers in each row and each column is the same. If exactly one number is changed in this picture, the result is a magic square. Which number must be changed?

22	1	16
9	13	19
10	25	4

3C *Time: 5 minutes*

Suppose the boxes are filled in with the digits from 0 through 6. Each digit is written exactly once. What three-digit number is the correct answer to the addition problem?

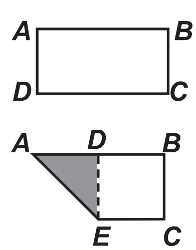


3D *Time: 7 minutes*

1 blue marble and 2 green marbles cost 16 cents.
1 red marble and 2 blue marbles also cost 16 cents.
1 green marble and 2 red marbles only cost 13 cents.
How much does 1 green marble cost?

3E *Time: 7 minutes*

The perimeter of rectangle *ABCD* is 36 cm. Suppose side *AD* is folded up so that *D* lies on the midpoint of side *AB* and $\overline{AE}$ is the crease as shown. What is the **area** of figure *ABCE*?



Please fold over on line. Write answers on back.

Division **E**

Mathematical Olympiads

FEBRUARY 7, 2012

for Elementary and Middle Schools

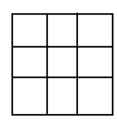
Contest **4**

4A *Time: 3 minutes*
What is the sum of the digits in the arrangement at the right?

4
4 3 4
4 3 2 3 4
4 3 2 1 2 3 4
4 3 2 1 0 1 2 3 4
4 3 2 1 2 3 4
4 3 2 3 4
4 3 4
4

4B *Time: 4 minutes*
Staci looks at the first and fourth pages of a chapter in her book. The sum of their page numbers is 47. On what page does the chapter begin?

4C *Time: 4 minutes*
The digits 1 through 9 are placed in the boxes shown, one per box. In each corner box is a prime number. In each box in the middle column is a square number. In the 3 boxes of the middle row is the least 3-digit number possible. What is that 3-digit number?



4D *Time: 7 minutes*
Different letters represent different digits. *AB* is an even 2-digit number. *EEE* is a 3-digit number. Find the 2-digit number *AB*.

$$\begin{array}{r} AB \\ M\overline{)EEE} \end{array}$$

4E *Time: 6 minutes*
A rectangular solid that is 4 cm by 6 cm by 8 cm is painted on all six faces. Then the solid is cut into cubes that measure 2 cm on each side. How many of these cubes have only one face painted?

Please fold over on line. Write answers on back.

5A ***Time: 3 minutes***

If the 5-digit number 3367N is divisible by 15, what is the digit N?

5B ***Time: 4 minutes***

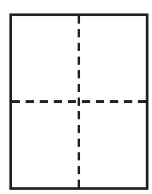
Vera makes vegetable trays. Each tray uses $\frac{2}{3}$ of a pound of carrots. Vera needs to make 25 trays. She buys carrots only in 2-pound bags. How many bags of carrots must Vera buy to make the 25 trays?

5C ***Time: 6 minutes***

At lunch, Hannah tells Dom, “If you give me 4 grapes, we will each have the same number of grapes.” Dom tells Hannah, “If you give me 4 grapes, I will have five times as many grapes as you will have then.” How many grapes does Hannah have?

5D ***Time: 6 minutes***

The perimeter of a rectangular piece of paper is 50 cm. It is cut into 4 congruent rectangles as shown. What is the total of the perimeters of the four smaller rectangles?



5E ***Time: 7 minutes***

AB, *CD*, *EF*, *GH*, and *JK* are five 2-digit numbers. Different letters represent different digits. Find the greatest possible value of the fraction below.

$$\frac{AB + CD + EF}{GH - JK}$$

Please fold over on line. Write answers on back.

Division E	Mathematical Olympiads NOVEMBER 15, 2011 for Elementary and Middle Schools	Contest 1

SOLUTIONS AND ANSWERS

1A **METHOD 1:** *Strategy: Find a pattern.*

(20 + 40 + 60 + 80 + 100 + 120)
– (10 + 30 + 50 + 70 + 90 + 110)
10 + 10 + 10 + 10 + 10 + 10. **The value is 60.**

METHOD 2: *Strategy: Perform the operations as indicated.*

20 + 40 + 60 + 80 + 100+ 120 = 420
10 + 30 + 50 + 70 + 90 + 110 = 360
Then 420 – 360 = 60. The value is 60.

1B **METHOD 1:** *Strategy: Start with an extreme case.*

Suppose all 20 creatures are owners. There would then be a total of 40 legs. The extra 24 legs must be accounted for by the dogs. Since each dog has 2 more legs than its owner, **there are** 24 ÷ 2 = **12 dogs in the group.**

METHOD 2: *Strategy: Set up a table and look for a pattern.*

The number of legs is 2 times the number of owners plus 4 times the number of dogs.

Number of owners	20	19	18	17	⋯	**?**
Number of dogs	0	1	2	3	⋯	**?**
Number of legs	40	42	44	46	⋯	**?**

Each increase of 1 in the number of dogs increases the number of legs by 2. To increase from 40 legs to 64 legs, a total of 24, requires an increase of 12 in the number of dogs. There are 12 dogs in the group.

METHOD 3: *Strategy: Use algebra.*

Let D = the number of dogs. Then 20 – D = the number of owners and 4D + 2(20 – D) = the number of legs. 4D + 2(20 – D) = 64. Solving, D = 12. There are exactly 12 dogs in the group.

FOLLOW–UP: *In a room there are three-legged stools and four-legged chairs. There are total of 14 seats and 47 legs. How many chairs are in the room?* [5]

1C **METHOD 1:** *Strategy: Determine how many times Keri came in first.*

Keri could not have finished first 3 or more times because that would give her more than 20 points. Suppose she finished first twice, a total of 16 points. Any remaining points must have come from finishing second, scoring 3 points each time. She can't have scored 4 points.

The same reasoning shows she can't have finished first 0 times since 20 is not a multiple of 3. So Keri finished first once, for 8 points. The remaining 12 points came from 4 second place finishes. She played 8 games, so **Keri finished last in** the remaining **3 games.**

Answers	
1A	60
1B	12
1C	3
1D	6
1E	2532

METHOD 2: *<u>Strategy</u>: List multiples of 8 and of 3 that are less than 20.*

Possible scores from first place finishes:	0, **8,** 16
Possible scores from second place finishes:	0, 3, 6, 9, **12,** 15, 18

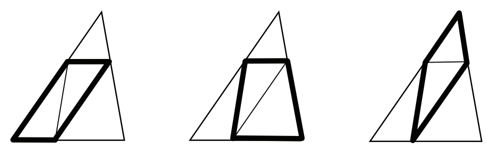
Only 8 and 12 sum to 20. This is 1 first place and 4 second places, so Keri finished last in 3 games.

> ***Follow-Ups:*** *(1) Suppose the winner of each game scored 5 points instead of 8. How many games might Keri have won?* [1 or 4]. *(2) Suppose the winner scored 6 points. Is it possible for Keri to earn 20 points?* [No! — why?]

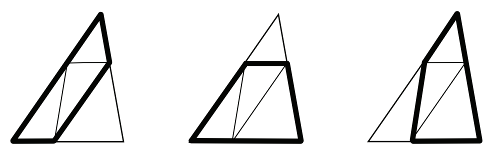
1D *<u>Strategy</u>: Count in an organized way.*

Four-sided figures can be formed by combining (*or by eliminating*) triangles in the picture.

Combine 2 triangles
(*or eliminate 2 triangles*):



Combine 3 triangles
(*or eliminate 1 triangle*):



6 four-sided figures can be traced.

1E *<u>Strategy</u>: Use reasoning and number properties.*

The sum of three 3-digit numbers must be less than 3000, so S is 1 or 2.

Suppose S = 1. In the ones column, D + D + D ends in 1 so D = 7. In the tens column, 7 + 7 + 7 + 2 ends in 3 so M = 3 and there is a "carry" of 2. In the hundreds column, A + A + A + 2 = 1U. A is even and can't be 2 (too small) or 6 or 8 (both too large). But if A = 4, U is also 4, and different letters represent different digits. There is no solution if S = 1.

Try S = 2. In the ones column, D + D + D ends in 2 so D = 4 and there is a carry of 1. In the tens column, 4 + 4 + 4 + 1 ends in 3 so M = 3, with a carry of 1. In the hundreds column, A + A + A + 1 = 2U. Only if A is 8 is the sum greater than 20, and then U = 5. **SUMS is 2532.**

> ***Follow-Up:*** *Different letters represent different digits. Find the sum CDD5 if ABB + BAB + BBA = CDD5. There are 2 solutions.* [1665 and 2775]

> ***NOTE:*** *Other Follow-Up problems related to some of the above can be found in our two contest problem books and in "Creative Problem Solving in School Mathematics."*
> *Visit <u>www.moems.org</u> for details and to order.*

SOLUTIONS AND ANSWERS

2A *Strategy: Make the smallest possible increase in the number.*
To keep a digit-sum of 3, move the 1 one place to the left. **The next larger number whose digit-sum is 3 is 2100.**

> ***Follow-Ups:*** *(1) How many 4-digit numbers have a digit-sum of 3?* [10 (1110, 1101, 1011, 1002, 1020, 1200, 2001, 2010, 2100, 3000)] *(2) How many 4-digit numbers have a digit-sum of 4?* [20]

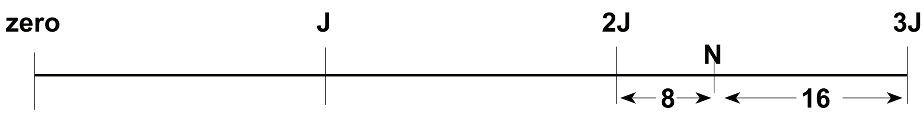
2A answer: 2100

2B *Strategy: Determine the number of letters in the repeating part.*
In the sequence A B B C C D A B B C C D . . . , there are 6 letters before the pattern repeats itself. The first 78 terms of the sequence contain 13 complete repetitions of the pattern and no additional letters. **The 78th letter is** the same as the 6th letter, **D.**

> ***Follow-Ups:*** *(1) In the sequence of letters ABBCCCDDDDEEEEEFFFFFF ..., what is the 100th letter?* [N] *(2) A letter is chosen in* ***Follow-Up 1*** *from the first 50 letters. What is the probability that it is a vowel?* [$^{15}/_{50}$ or $^{3}/_{10}$.]

2B answer: D

2C **METHOD 1:** *Strategy: Draw a diagram.*
On the number line, point *J* represents Jen's age, point 2*J* represents twice her age and point 3*J* represents three times her age. Point *N* represents the number that is both 8 more than 2*J* and 16 less than 3*J*.



The distance from 2*J* to 3*J* is the same as the distance from zero to *J*; that is Jen's age. Thus **Jen's age is** 8 + 16 = **24.**

METHOD 2: *Strategy: Use algebra.*
The variables *J*, 2*J*, 3*J*, and *N* are as defined above.
Then $N = 3J - 16 = 2J + 8$.
Solving, $J = 24$.
Jen's age is 24.

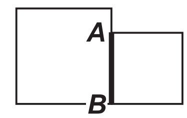
2C answer: 24

2D answer: 44

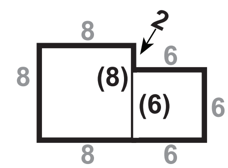
2E answer: 56

2D *Strategy: Find the areas of the two squares.*
List the square numbers: 1,4,9,16,25,36,49,64,81,100. Only 36 and 64 add to 100. Then the larger square is 8 cm on each side and the smaller is 6 cm on each side. Now find the perimeter of the figure.

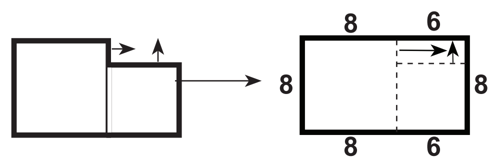
METHOD 1: *Strategy: Find the total perimeter of the 2 squares, and adjust.*
The total perimeter of the squares is 4 × 8 + 4 × 6 = 56 cm. But this counts the length of segment *AB* twice, once as part of each square. It should not be counted at all. **The perimeter of the figure is** 56 – 2 × 6 = **44 cm.**



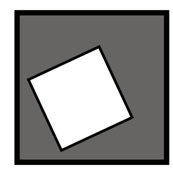
METHOD 2: *Strategy: Find the length of each segment.*
The two squares have 6 cm in common, so the larger square contributes an extra 8 – 6 = 2 cm to the perimeter. Then the perimeter of the figure is 2 + (3 × 8) + 3 × 6 = 44 cm.



METHOD 3: *Strategy: Slide segments to make the figure into a rectangle.*
The diagram on the left indicates the two segments to be moved. The resulting diagram on the right indicates that the same lengths will be added to find the perimeter, but in a more convenient fashion. Thus, the perimeter of the figure is (2 × 14) + (2 × 8) = 44 cm.



Follow-Ups: *(1) 2 squares whose side-lengths are whole numbers are placed so that one is entirely inside the other. If the area of the region between the two squares is 45, what are two possible pairs of lengths of the sides?* [(7,2), (9,6) are the most readily found, but (23,22) also works.]



2E *Strategy: Work backwards.*
The table below shows actions in reverse. The first column names each person, last to first. The second column shows the actions each person took. The third column states the number of coins on the table as each person approached.

NAME	ACTION	MUST HAVE STARTED WITH
Selena	Took 5, left 4.	9 coins
Joe	Took 2, left 9	11
	Took half, left 11	22
Nick	Took 2, left 22	24
Sara	Took 4, left 24	28
	Took half, left 28	56

56 coins were on the table to start with.

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Division **E** | **Mathematical Olympiads** JANUARY 10, 2012 *for Elementary and Middle Schools* | Contest **3**

SOLUTIONS AND ANSWERS

3A *Strategy: Make a list.*
If the ones digit is twice the tens digit, the only possibilities are 12, 24, 36, and 48. Tens digits of 5 through 9 would produce two digits in the ones place; zero can't be a leading digit. If the tens digit is twice the ones digit, the only possibilities are 21, 42, 63, and 84. Thus, **8 two-digit numbers have one digit that is twice the other.**

3B *Strategy: Find the sum of each row and each column.*
If only one number needs to be changed, it must sit in a row and column that has a different sum from the others. The diagram shows the sums of each row and column.

columns

rows				
	22	1	16	**39**
	9	13	19	**41**
	10	25	4	**39**
	41	**39**	**39**	

The middle row has a different sum from the other two. The sum of the first column is different from the other two. **The number that must be changed is** in the middle row, first column, **9.** (If the 9 is replaced by 7, the result is a magic square.)

3C *Strategy: Work from left to right.*

$$\begin{array}{r} \mathbb{A}\ \mathbb{B} \\ +\ \ \mathbb{C}\ \mathbb{D} \\ \hline \mathbb{E}\ \mathbb{F}\ \mathbb{G} \end{array}$$

Use the letters $\mathbb{A}$ through $\mathbb{G}$ to designate the seven digits.

$\mathbb{E}$ = 1, since the sum of two 2-digit numbers is less than 200.

This means that the tens digits must add up to 10 (or 9 plus a carry from the ones column).

There are 4 cases for the tens column:
- 6 and 5 plus a carry, because $\mathbb{F}$ and $\mathbb{E}$ cannot both be 1;
- 6 and 4;
- 6 and 3 plus a carry; and
- 5 and 4 plus a carry.

In all three cases in which the units place needs to produce a carry, the remaining digits are too small to do so. This leaves 6 and 4 in the tens place, and the last column must add 2 and 3 to get 5. **The answer to the addition problem is 105.**

Problem	Answer
3A	**8**
3B	**9**
3C	**105**
3D	**5**
3E	**54**

3D **METHOD 1:** *<u>Strategy</u>: Set up a table. Use number properties to limit choices.*
The cost of 1 green and 2 red marbles is an odd number of cents but the cost of 2 red marbles is even. Then the cost of 1 green marble is odd.

Suppose 1 green costs	3¢	**5¢**	7¢	9¢
Then 2 red = 13¢ – 1 green, and	10	**8**	6	4
Therefore, 1 red costs	5¢	**4¢**	3¢	2¢
Next, 2 blue = 16¢ – 1 red	11	**12**	13	14
Therefore, 1 blue costs	——	**6¢**	——	7¢
CHECK: 1 blue + 2 green costs	——	**16**	——	25

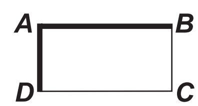
A green marble costs 5 cents.

METHOD 2: *<u>Strategy</u>: Combine the given information.*
Suppose all 3 purchases are made. Then 3 green marbles, 3 blue marbles, and 3 red marbles cost 45 cents. So 1 green marble, 1 blue marble, and 1 red marble cost 15 cents.

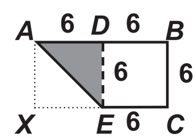
The first and second sentences show that 1 blue and 2 green marbles cost as much as 1 red and 2 blue marbles. If we imagine a balance scale with 1 blue and 2 green marbles on one side and 1 red and 2 blue marbles on the other, we could remove 1 blue marble from each side and see that 2 green marbles balance 1 red and 1 blue marble. Thus, we could replace 1 red and 1 blue with 2 green marbles. So we know that 3 green marbles cost 15 cents, so each green marble costs **5 cents**.

FOLLOW-UP: *At a movie theater, 2 popcorns and a soda cost $13, while 5 popcorns and 4 sodas cost $37. Julia orders a popcorn and a soda. How much does Julia spend?* [$8]

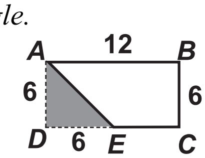
3E **METHOD 1:** *<u>Strategy</u>: Add the areas of triangle* ADE *and square* DBCE.
The *semi*-perimeter of the rectangle (that is *DA* + *AB*) is 18 cm.



After folding, $\overline{AD}$ lies on $\overline{AB}$ with *D* touching the midpoint of $\overline{AB}$. Then *AB* is twice *AD*, and the original rectangle is 12 cm by 6 cm. Then *DBCE* is a 6 cm by 6 cm square and its area is 36 sq cm. Look at triangle *ADE*: it actually is half of square ADEX, also a 6 cm by 6 cm square; its area is half of 36 sq cm = 18 sq cm. **The area of trapezoid *ABCE* is** 36 + 18 = **54 sq cm.**



METHOD 2: *<u>Strategy</u>: Subtract the area of the shaded region from the rectangle.*
As before, the original rectangle is 12 cm by 6 cm. Its area is 72 sq cm. *AD* and *DE* are each 6 cm long and the area of the shaded region (again half of a 6 × 6 square) is 18 sq cm. Then the area of *ABCE* is 72 – 18 = 54 sq cm.



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Division **E**

Mathematical Olympiads

FEBRUARY 7, 2012

for Elementary and Middle Schools

Contest **4**

SOLUTIONS AND ANSWERS

4A *Strategy: Count how many times each digit appears.*
There are sixteen 4s, twelve 3s, eight 2s, four 1s, and one 0.
The sum of the digits is (16×4) + (12×3) + (8×2) + (4×1) =
64 + 36 + 16 + 4 = **120.**

```
        4
      4 3 4
    4 3 2 3 4
  4 3 2 1 2 3 4
4 3 2 1 0 1 2 3 4
  4 3 2 1 2 3 4
    4 3 2 3 4
      4 3 4
        4
```

FOLLOW-UPS: *(1) Suppose the diagram above is surrounded by a border of 5s. What is the sum of the digits in the new picture?* [220] *(2) Suppose the pattern is continued with a border of 6s, then 7s, and then 8s. What is the sum of the 8s in the new picture?* [256]

4B **METHOD 1:** *Strategy: Find the average of the page numbers.*
The sum of the first and fourth page numbers is 47. This is also the sum of the second and third page numbers. The average of the 4 page numbers is 23½. 23½ is immediately between the second and third page numbers. The page numbers are 22, 23, 24, and 25. **The chapter begins on page 22.**

METHOD 2: *Strategy: Use algebra.*
Let P represent the first page number. The other numbers are $P + 1$, $P + 2$, and $P + 3$. Then $P + (P + 3) = 47$. Solving, $P = 22$. The chapter begins on page 22.

FOLLOW-UPS: *(1) Max just finished reading 7 consecutive pages for homework. The sum of the page numbers he read is 392. What page numbers did he read?* [pp. 53-59] *(2) Four brothers are born one year apart. The sum of their ages is the father's age which is two less than five times the youngest's age. How old is the father?* [38]

4C *Strategy: List the digits that satisfy each condition.*
Place the prime numbers {2, 3, 5, 7} in the corner boxes.

Place the squares {1, 4, 9} in the middle column.

Place the remaining digits {6, 8} in the remaining boxes of the middle row.

Read the middle row, left to right, and choose the least unused number in each box. **The least number is 618.**

2,3,5,7	1,4,9	2,3,5,7
6,8	1,4,9	6,8
2,3,5,7	1,4,9	2,3,5,7

FOLLOW-UPS: *What is the least three-digit number that can be formed if the first digit is prime, the second digit is square, the third digit is even, and no digit is repeated?*[204]

Problem	Answer
4A	120
4B	22
4C	618
4D	74
4E	4

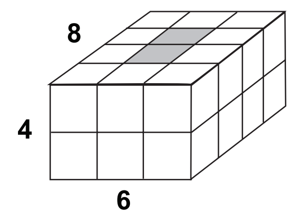
4D **METHOD 1:** *<u>Strategy</u>: Find the factors.*
Write the problem as a multiplication: M × AB = EEE. Any number of which all three digits are the same is a multiple of 111. 111 is a multiple of 3 because its digit-sum is 3. The prime factors of 111 are 3 and 37. AB must be a multiple of 37, inasmuch as M is only one digit. Since AB is an even 2-digit number, **AB is** 2 × 37 = **74**.

METHOD 2: *<u>Strategy</u>: Use number properties to reduce the possible guesses.*
As above, EEE is a multiple of 3. Further, since M × AB = EEE and AB is even, EEE = 222, 444, 666, or 888. Divide each by 3, 6, and 9: 222 ÷ 3, 444 ÷ 6 and 666 ÷ 9 each produce a quotient of 74. 222 ÷ 6 = 37, which is not even. All other choices produce a nonzero remainder or a three-digit quotient. The only even possible value for AB is 74.

Follow-Ups: *(1) Find C if AB × C = AAA.* [9] *(2) Replace each letter with a different digit to make the division at the right correct.* [BET=247, THAT=7657, ON=31]

```
        ON
BET ) THAT
      TEN
        BET
        BET
```

4E *<u>Strategy</u>: Draw a diagram.*
Draw the rectangular solid showing how it was cut into 2 cm cubes. Eliminate the 8 corner cubes (3 faces painted) and the 12 edge cubes (2 faces painted.) **4 of these cubes have only one face painted.**



Follow-Ups: *Suppose the rectangular solid in 4E is cut into 1-cm cubes. (1) How many cubes have three faces painted?* [8] *(2) No faces painted?* [48] *(3) Into how many 1-cm by 2-cm by 3-cm rectangular solids can the figure in 4E be cut?* [32]

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SOLUTIONS AND ANSWERS

Problem	Answer
5A	5
5B	9
5C	8
5D	100
5E	222

5A **METHOD 1:** *Strategy: Use divisibility rules for 3 and 5.*
A number is divisible by 15 if it is divisible by both 3 and 5. Because the number 3367N is divisible by 3, its digit-sum is divisible by 3. Thus, 19 + N is divisible by 3. If a number is divisible by 5, its units digit is 0 or 5. If N = 0, 19 + 0 = 19 is not divisible by 3. If N = 5, 19 + 5 = 24 is divisible by 3, so **the digit N = 5**.

METHOD 2: *Strategy: Do the division.*

```
     . 2 2 4 ?
15 )3 3 6 7 N
    3 0
      3 6
      3 0
        6 7
        6 0
          7 N
```

7N must be divisible by 15. Since 75 = 15×5, N is 5.

Follow-Up: *If the number 51A6B is divisible by 36, what numbers could 51A6B represent?* [51660 or 51264 or 51768]

5B *Strategy: First find the number of pounds needed.*
Each tray needs $^2/_3$ pound of carrots, so 25 trays need $25 \times {}^2/_3 = {}^{50}/_3$ or $16^2/_3$ pounds. Since carrots come in 2-pound bags, 8 bags won't suffice, so **Vera must buy 9 bags of carrots**.

5C **METHOD 1:** *Strategy: Set up a table of the differences in their amounts.*
Each has at least 4 grapes.

If Dom gives Hannah 4 grapes, she gains 4 and he loses 4. If their totals become equal, he must have 8 more grapes than she.

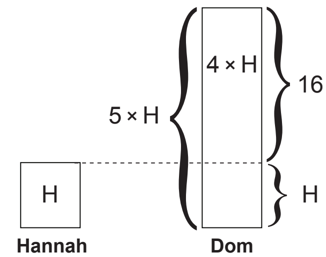
In this table, the first two columns show how many grapes each could have now: 12 and 4, 13 and 5, 14 and 6, and so on. The other two columns show the results in each case if Hannah gives Dom 4 grapes.

Current Status		Hannah gives Dom 4 grapes	
Dom	Hannah	Dom	Hannah
12	4	16	0
13	5	17	1
14	6	18	2
15	7	19	3
16	8	**20**	**4**

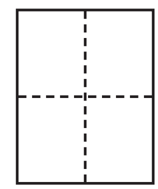
Since 20 = 5 × 4, **Hannah** really **has 8 grapes** and Dom 16.

METHOD 2: *Strategy: Use a diagram to show the results.*
As in method 1, Dom has 8 more grapes than Hannah.

If she gives him 4 grapes, he would have 16 more than she, by the same reasoning as in method 1. The diagram at the right shows why the 16 is four times her final quantity. Then her final quantity would be 4 grapes. Thus, when they speak Hannah has 8 grapes.

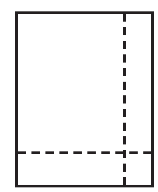


5D **METHOD 1:** *Strategy: Find the perimeter of 1 small rectangle.*
Each small rectangle has a length that is half the length of the large rectangle and a width that is also half as large. The perimeter of a small rectangle is then half the perimeter of the large rectangle, 25 cm. **The total of the perimeters of the 4 smaller rectangles is** 4×25 = **100 cm.**



METHOD 2: *Strategy: Assign numerical values to the length and width.*
The perimeter of the paper is 50 cm, so the sum of length and width is 25 cm. Suppose the length is 20 cm and the width is 5 cm. Each small rectangle then has a length of 10 cm and a width of 2½ cm. The perimeter of each small rectangle is 20 + 5 = 25 cm, and the total of the four perimeters is 100 cm.

Follow-Up: *(1) Suppose one cut were 8 cm lower and the other 3 cm to the right as shown. How would the sum of the perimeters be affected?* [It wouldn't.] *(2) The perimeter of a checkerboard is 100 cm. What is the sum of the perimeters of its 64 squares?* [800 cm]



5E *Strategy: Minimize the denominator. Then maximize the numerator.*
The smallest possible denominator is 1, which can be obtained by using 20 – 19, 30 – 29, etc. To save as many large digits as possible for the numerator, use 20 – 19 for the denominator. The digits remaining are those from 3 through 8. To make the numerator as large as possible, use 8, 7, and 6 as the tens digits (in any order) and 3, 4, and 5 as the ones digits (in any order). **The greatest possible value is** $\frac{83 + 74 + 65}{20 - 19}$ = **222.**

Follow-Up: *What is the least possible positive value of the fraction?* [$\frac{27}{22}$]

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