Olympiad Problems

2010-2011

Division E

WITH ANSWERS AND SOLUTIONS

MATH OLYMPIADS Since 1979

Our Thirty-SecondYear

Mathematical Olympiads for Elementary and Middle Schools

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1A *Time: 3 minutes*

Suppose it is now 4:00 PM. What time will it be in 245 hours? Label your answer AM or PM.

1B *Time: 4 minutes*

Ashley's locker number is a three-digit multiple of 5. The tens digit is the sum of the hundreds digit and the ones digit. The sum of all three digits is 16. No digit is repeated. What is Ashley's locker number?

1C *Time: 5 minutes*

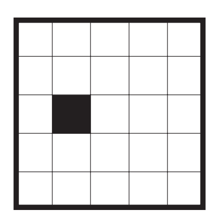
Ten friends have an average of 5 toy soldiers each. Lee joins them, and now the average is 6 toy soldiers each. How many toy soldiers does Lee have?

1D *Time: 6 minutes*

Tracy's Trophies charges by the letter for engraving. There is one fee for each vowel and a different fee for each consonant. CAROL costs $31 to engrave. GABRIEL costs $43 to engrave. How many dollars does BRIDGET cost to engrave?

1E *Time: 6 minutes*

As shown, the 5 × 5 "checkerboard" contains one shaded square. In this diagram, how many squares <u>of any size</u> do not include the shaded square?



2A ***Time: 3 minutes***
What number does N represent?

$$10 + 20 + 30 + 40 + 50 + N = 220$$

2B ***Time: 4 minutes***
What is the least multiple of 9 that is greater than 150?

2C ***Time: 5 minutes***
There are 2 red cars and 3 blue cars. The 5 cars contain a total of 12 people. No car has more than 4 people. Every car has at least 1 person. The only cars with the same number of people are the red cars. How many people are in **1 red car**?

2D ***Time: 5 minutes***
What number does **G** represent in the following?
A baseball team won $\frac{3}{4}$ of its first 24 games. Then the team lost its next **G** games. As a result, the team had now won-exactly half of its games.

2E ***Time: 7 minutes***
The first number on a list has two digits. The second number on the list is the first number plus the sum of its digits. The third number on the list, 44, equals the second number plus the sum of *its (the second number's)* digits. What is the first number?

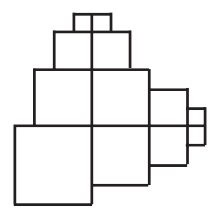
3A *Time: 3 minutes*
I am thinking of a number. If you subtract 3 from my number and then multiply by 4, the result is 28. What number am I thinking of ?

3B *Time: 4 minutes*
What number between 104 and 140 is exactly divisible by 6 and exactly divisible by 15?

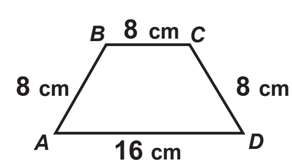
3C *Time: 6 minutes*
At the end of a power outage, a digital clock resets to 12:00 midnight. At 9:35 AM on the same day as the power outage occurred, the digital clock shows 3:50 AM. At what time did the power outage end?

(Label your answer AM or PM)

3D *Time: 6 minutes*
The figure shown is built from four 1x1-squares, four 2x2-squares, three 3x3-squares, and one 4x4-square (each measured in cm). What is the perimeter of the figure, in cm?



3E *Time: 6 minutes*
Two bugs walk from point *A* to point *D* along the sides of figure *ABCD*. They start and finish together. The first bug walks from *A* to *B* to *C* to *D* at an average speed of 3 centimeters per second. The second bug walks directly from *A* to *D*. What is the average speed of the second bug?



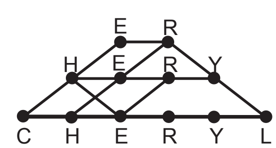
4A ***Time: 3 minutes***
Zach buys two hot dogs and three drinks for \$14. Drinks cost \$2 each. How much does one hot dog cost?

4B ***Time: 5 minutes***
Michael has \$5 less than Samantha. Samantha has \$10 more than Rob. Rob has \$15 less than Hailey. How many more dollars does Hailey have than Michael?

4C ***Time: 5 minutes***
A list contains exactly 6 different counting numbers. No number in the list is a multiple of any other in the list. What is the least possible total of these 6 numbers?

4D ***Time: 7 minutes***
Each of Mia's marbles has several colors on it. $\frac{2}{5}$ of the marbles have some red on them. $\frac{3}{4}$ of the marbles have some yellow on them. $\frac{6}{7}$ of the marbles have some blue on them. Mia has fewer than 250 marbles. How many of Mia's marbles have some blue on them?

4E ***Time: 7 minutes***
Cheryl traces her name, **CHERYL,** by following the lines shown. She can change direction only at a letter. How many different paths can trace her name?



5A ***Time: 5 minutes***

How many digits are in the product of the following:

2 x 3 x 5 x 2 x 3 x 5 x 2 x 3 x 5?

5B ***Time: 4 minutes***

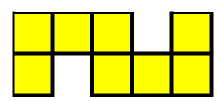
Lisa has a secret 3-digit number. No digit is 0. She says:

The hundreds digit is a multiple of 4.
The tens digit is a perfect square.
The ones digit is a multiple of 3.
The digits are in decreasing order.

What is Lisa's secret number?

5C ***Time: 5 minutes***

The figure shown consists of 8 identical squares. The area of the figure is 8 square centimeters. What is the perimeter of the figure, in cm?



5D ***Time: 7 minutes***

How many 3-digit numbers have exactly 2 digits that are the same?

5E ***Time: 7 minutes***

A broken clover has 2 leaves, a normal clover has 3 leaves, and a lucky clover has 4 leaves. In a bunch of these clovers, there are twice as many normal clovers as broken clovers and 5 times as many normal clovers as lucky clovers. The bunch has a total of ***N*** leaves. ***N*** is greater than 200. What is the least value of ***N***?

Note: Number in shaded rectangle indicates percent of all competitors with a correct answer.

OLYMPIAD 1 NOVEMBER 16, 2010

Answers: [1A] **9PM** [1B] **385** [1C] **16** [1D] **45** [1E] **39**

44% correct

1A *Strategy: Find the elapsed time in days.*
Every 24 hours, the time is again 4:00 PM. 245 ÷ 24 = 10 R5. **In 245 hours it will be** 10 days and 5 hours later, or **9 PM.**

FOLLOW-UP: *If it is noon now, what time was it 161 hours and 27 minutes ago?* [6:33 PM]

55%

1B **METHOD 1:** *Strategy: Find the ones digit first.*
Ashley's locker number ends in 0 or 5. It cannot be 0 because then the hundreds digit and the tens digit would be the same. The only other choice for the ones digit is 5. Since the tens digit is 5 more than the hundreds digit, her locker number is one of the following: 165, 275, 385, 495. Of these, only 385 has a digit sum of 16. **Ashley's locker number is 385.**

METHOD 2: *Strategy: Find the tens digit first.*
The sum of the digits is 16 and the tens digit is the sum of the other 2 digits, so the tens digit is half the sum, 8. As above, the ones digit is 0 or 5, giving possibilities of 880 or 385. The digits must be different, so Ashley's locker number is 385.

31%

1C **METHOD 1:** *Strategy: Use the definition of an average.*
The ten friends have a total of 10 × 5 = 50 toy soldiers. When Lee joins the group, the 11 friends have a total of 11 × 6 = 66 toy soldiers. Then **Lee has** 66 – 50 = **16 toy soldiers.**

METHOD 2: *Strategy: Increase each friend's total by the same amount.*
Suppose Lee also has 5 soldiers. The average for all 11 friends would still be 5. To increase the average to 6, add 1 to each of the 11 friends totals. Adding 11 to the total number of soldiers is equivalent to adding 11 to Lee's total. Lee has 16 toy soldiers.

FOLLOW-UP: *On a math test, the average of the 8 girls in the class was 85, and the average of the 12 boys in the class was 80. What was the average of all the students in the class?* [82]

28%

1D **METHOD 1:** *Strategy: Compare the given names.*
GABRIEL has 1 more consonant and 1 more vowel than CAROL. Then 1 consonant and 1 vowel together cost 43 – 31 = $12. 2 consonants and 2 vowels together cost twice as much, 2 × 12 = $24. CAROL has 3 consonants and 2 vowels. So 1 consonant costs 31 – 24 = $7. Then the 3 consonants in CAROL cost $21, and the 2 vowels cost 31 – 21 = $10. 1 vowel costs $5. **BRIDGET**, with 5 consonants and 2 vowels, **costs** 5×7 + 2×5 = **$45 to engrave.**

METHOD 2: *Strategy: Make a chart. Use number properties to limit the guesses.*
CAROL, with 3 consonants and 2 vowels, costs \$31, an odd number. Since the cost of 2 vowels must be even, the cost of 3 consonants must be odd, and therefore the cost of 1 consonant is odd. The table below tests different odd costs for a consonant to see which one produces the \$43 cost for GABRIEL.

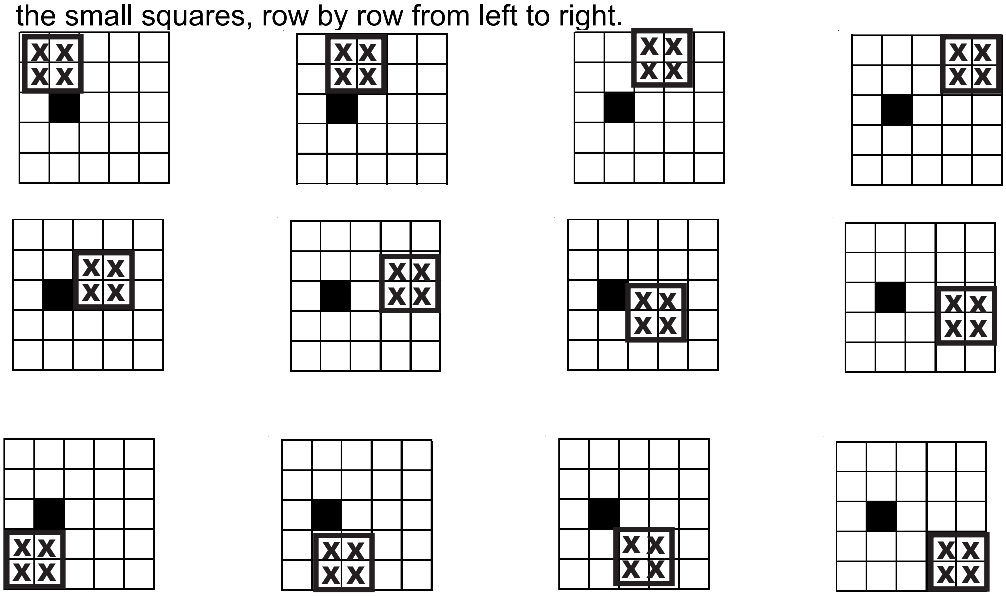
Then each consonant costs \$7, each vowel costs \$5, and BRIDGET costs \$45 to engrave.

COST IN DOLLARS FOR:						
one consonant	**CAROL**		***one vowel***	**GABRIEL**		
	CRL	AO		GBRL	AIE	GABRIEL
\$3	9	31– 9 = 22	\$11	4×\$3 = 12	3×11 = 33	12+33 = 45
\$5	15	31–15 = 16	\$8	4×\$5 = 20	3×8 = 24	20+24 = 44
\$7	**21**	**31–21 = 10**	**\$5**	**4×\$7 = 28**	**3×5 = 15**	**28+15 = 43**

12%

1E *Strategy: Count in an organized way.*
1×1 squares: 24 of the 25 squares have no shading.

2×2 squares: The following diagrams show the result of counting the possible positions of the small squares, row by row from left to right.



Twelve 2×2 squares do not include the shaded square
3×3 squares: Three 3x3 squares do not include the shaded square.
4×4 squares: None: all of the 4x4 squares include the shaded square.
In all **there are 39 squares in the diagram that do not include the shaded square.**

FOLLOW-UP: On a 5 × 5 unshaded checkerboard, how many 1 × 2 rectangles are there? [40]

OLYMPIAD 2 DECEMBER 14, 2010

Answers: [2A] **70** [2B] **153** [2C] **2** [2D] **12** [2E] **29**

91% correct

2A *<u>Strategy</u>: Simplify the expression.*
10 + 20 + 30 + 40 + 50 = 150. 150 +70 = 220, so ***N* represents 70.**

58%

2B METHOD 1: *<u>Strategy</u>: Divide and find the remainder.*
150 ÷ 9 = 16 R6. Since 150 is 6 more than a multiple of 9, it is also 9 – 6 = 3 less than the next multiple of 9. Then 150 + 3 = **153 is the least multiple of 9 that is greater than 150.**

METHOD 2: *<u>Strategy</u>: List the multiples of 9.*
Start with a known multiple of 9. Continue adding 9 until the sum first exceeds 150. For example: 99, 108, 117, 126, 135, 144, 153. 153 is the least multiple of 9 greater than 150.

METHOD 3: *<u>Strategy</u>: Use the digit sum.*
A multiple of 9 has a digit sum that is also a multiple of 9. The digit sum of 150 = 1 + 5 + 0 = 6, so 3 must be added to get a multiple of 9. 153 is the least multiple of 9 greater than 150.

***FOLLOW-UP:** What is the greatest multiple of 8 less than 150?* [144]

46%

2C METHOD 1: *<u>Strategy</u>: Start with a different number of people in each car.*
If the number of people in every car is different and no car is empty, the minimum number of people would be 1 + 2 + 3 + 4 + 5 = 15. But there are only 12 people, and no car has 5 people. Take 3 of the 5 people out of that last car, leaving 1 + 2 + 3 + 4 + 2 = 12 people. Only the two red cars have the same number, so **each red car has 2 people.**

METHOD 2: *<u>Strategy</u>: Start with 1 person in each car.*
First place 1 person, the driver, in each car. The 5 cars must contain the remaining 7 people, with the 2 red cars having the same number of people. There are two ways to place the 7 people: {0,0,1,2,4} and {0,1,1,2,3}. Using {0,0,1,2,4} people would place 1, 1, 2, 3, and 5 people in the cars, but then 5 people would be in one car. Using {0,1,1,2,3} places 1, 2, 2, 3, and 4 people in them. The blue cars contain 1, 3, and 4 people, and each red car has 2 people.

***FOLLOW-UPS:** In how many ways can 5 identical tee shirts be distributed among 3 students: (1) if each student gets at least 1 tee shirt? (2) if some students might not get any tee shirts? [6, 21]*

41%

2D METHOD 1: *<u>Strategy</u>: Find the total number of games won.*
The team won $\frac{3}{4}$ of its first 24 games. $\frac{3}{4}$ x 24 = 18. The team won 18 and therefore lost 6 of its first 24 games. If in the end the team has won just half of its games, the team must have lost just as many games as it won, 18. Thus G + 6 =18 and **G represents 12.**

METHOD 2: *<u>Strategy</u>: Work with the fraction.*

Represent the first 24 games: $\underbrace{\boxed{W}\boxed{W}\boxed{W}}_{\frac{3}{4}}\underbrace{\boxed{L}}_{\frac{1}{4}} = 24$

Thus each box represents 6 games.

Now represent all the games: $\boxed{W}\boxed{W}\boxed{W}\boxed{L}\boxed{L}\boxed{L}$ The shaded boxes represent the G games. Since each box represents 6 games, G represents 12.

27%

2E **METHOD 1:** *Strategy: Form sequences (lists) using the given rule.*
Beginning with 10, 11, 12, etc., write out possible sequences and look for one that contains 44. Begin each new sequence with the least two-digit number that doesn't appear in a previous sequence.

Start	Sequence	44?
10	10, 11, 13 (11+2), 17, 25, 32, 37, **47**, …	*no*
12	12, 15, 21, 24, 30, 33, 39, **51**, …	*no*
14	14, 19, **29, 40, 44**, …	**YES**

Because 44 is the third number in the sequence, **the first number is 29.**

METHOD 2: *Strategy: Work backwards.*
Try a few numbers. Since 40 + 4 = 44, the second number is 40.

The first number plus the sum of its digits is 40. Choose a few numbers in the thirties. Notice in each case that adding the number to the sum of its digits produces an odd number. Therefore, start with 29. In fact, 29 + 11 = 40, so the first number is 29.

FOLLOW-UPS: (1) The first numbers in the Fibonacci Sequence are 1, 1, 2, 3, 5, 8, and 13. What is the sum of the first ten numbers in the sequence? [143] *(2) Other than 1, what is the least number in the Fibonacci Sequence that is a perfect square?* [144] *(3) Consider the answers to FOLLOW-UPS (1) and (2). In the Fibonacci Sequence, what pattern emerges as you compare the sums of the terms to the terms themselves?* [Each sum is 1 less than the Fibonacci Number two terms later.]

OLYMPIAD 3 — JANUARY 11, 2011

Answers: [3A] **10** [3B] **120** [3C] **5:45AM** [3D] **40** [3E] **2**

84% correct

3A **METHOD 1:** *Strategy: Work backwards, using opposite operations.*

Question: $\boxed{\textit{My Number}} \xrightarrow{-3} \boxed{?} \xrightarrow{\times 4} \boxed{28}$

Solution: $\boxed{28} \xleftarrow{+3} \boxed{7} \xleftarrow{\div 4} \boxed{28}$

My number is 10.

METHOD 2: *Strategy: Use algebra.*
Let N represent my number.

Then
$$4(N-3) = 28$$
$$N-3 = 7$$
$$N = 10.$$
The number is 10.

FOLLOW-UP: (1)When you multiply my number by 4, then divide by 2, then multiply by 4 and then divide by 8, the answer is 1. What is my number? [1]
(2) (A mental arithmetic challenge) Without using a calculator or paper, find the product of 99 x 43. [100 x 43 – 43 = 4257]

73%

3B METHOD 1: *Strategy: Use the Least Common Multiple (LCM).*
The LCM of 6 and 15 is 30. Because the only multiple of 30 between 104 and 140 is 120, **the number is 120**,

METHOD 2: *Strategy: Use divisibility rules.*
Multiples of 15 end in 0 or 5. Multiples of 6 are even. Then the number ends in 0. The possibilities are 110, 120, or 130. Any multiple of 6 is a multiple of 3, and so the sum of its digits is a multiple of 3. Of the three numbers listed, only 120 has a digit sum that is a multiple of 3. The number is 120.

METHOD 3: *Strategy: Use a table to compare multiples of 6 and 15.*
Consider only the multiples that are greater than 104.

Multiples of 15	105	**120**	135			
Multiples of 6	108	114	**120**	126	132	138

In the given interval only 120 is divisible by both 6 and 15.

Variation: List the multiples of the greater number, 15, that are in the interval and test each for divisibility by the lesser number, 6.

30%

3C The clock shows that the power outage ended 3 hours and 50 minutes ago. To determine the time, subtract 3 hours and 50 minutes from 9:35.
METHOD 1: *Strategy: Use a convenient time, then adjust.*
4 hours earlier, the time was 5:35. 3 hours and 50 minutes ago was 10 minutes later than that, so **the power outage ended at 5:45 AM.**

METHOD 2: *Strategy: Regroup.*

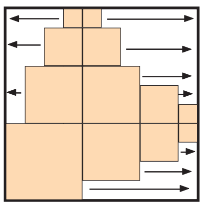
9 hours 35 minutes	8 hours 95 minutes
– 3 hours 50 minutes	– 3 hours 50 minutes
	5 hours 45 minutes.

The power outage ended at 5:45 AM.

FOLLOW-UP: (1) At 3:00 pm a clock shows the correct time. If it loses 12 minutes every hour, in how many hours will it next show the correct time? [60 hours]

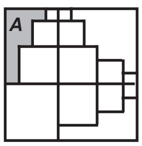
30%

3D METHOD 1: *Strategy: Change the figure to a simpler one with the same perimeter.*
Sliding segments to different locations does not change their lengths. Slide the vertical sides (or segments of sides) left or right as shown. Similarly, slide the horizontal segments to the top or bottom of the figure. This creates a square that is 10 cm on a side. The perimeter of the square, and therefore **the perimeter of the original figure, is 40 cm.**



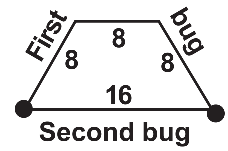
METHOD 2: *Strategy: Add the lengths of the sides of the figure.*
Note that each side of the figure that is not the side of one of the squares is 1 cm in length. Starting at the bottom left and going clockwise, the perimeter of the figure is 4+1+3+1+2+1+1+1+1+1+1+2+1+1+2+1+1+1+1+1+1+2+1+3+1+4 = 40 cm.

FOLLOW-UP: What is the area of region A in this diagram?
[$4 \times 6 - 1^2 - 2^2 - 3^2 = 10$ sq cm; other methods are possible.]



49%

3E **METHOD 1:** *Strategy: Compare the distances the two bugs walked.*
The second bug covers 16 cm in the same time the first bug covers 24 cm. Because the second bug covers only $^2/_3$ the distance of the first bug, it travels at $^2/_3$ the speed. The speed of the first is 3 cm per sec. **The speed of the second bug is 2 cm per sec.**



METHOD 2: *Strategy: Find the time that each bug traveled.*
The first bug needed 8 seconds to travel the 24 cm at the rate of 3 cm per second. The second bug traveled 16 cm in the same 8 seconds, so its rate was 2 cm per second.

OLYMPIAD 4 — FEBRUARY 8, 2011

Answers: [4A] **4** [4B] **10** [4C] **41** [4D] **120** [4E] **9**

85% correct

4A **METHOD 1:** *Strategy: Subtract the cost of the drinks*
Three drinks cost \$6. Therefore 2 hot dogs cost \$8. **One hot dog costs \$4.**

METHOD 2: *Strategy: Use algebra*
Let x represent the price of a hot dog. Then

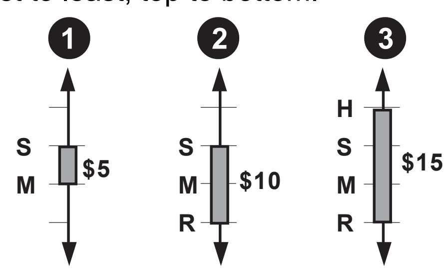
$2x + 3(2) = 14$

Solving, $x = 4$. One hot dog costs \$4.

63%

4B **METHOD 1:** *Strategy: Draw a diagram to compare their amounts.*
Represent each person by an initial. Rank from most to least, top to bottom.

1. Michael has \$5 less than Samantha.
2. Samantha has \$10 more than Rob.
3. Rob has \$15 less than Hailey.



The picture shows that **Hailey has \$10 more than Michael.**

METHOD 2: *Strategy: Assign a convenient amount to Michael.*
Suppose Michael has \$50. Then Samantha has \$55. Rob has \$45 and Hailey has \$60. Hailey has \$10 more than Michael.

FOLLOW-UP: In Method 2, choose other amounts for Michael. Why does this work?

30%

4C *<u>Strategy</u>: Add 1 number at a time to the list, starting with the least.*
Omit 1 from the list since every number is a multiple of 1. Put 2 on the list. Omit all multiples of 2 greater than 2. Put 3 on the list and omit all its multiples greater than 3. Put 5 on the list and omit all its multiples greater than 5. Continue the process until the list contains 6 numbers. *In this case* they happen to be the first 6 primes: 2, 3, 5, 7, 11, and 13. **The least possible total is 41.**

> ***Follow-Up:*** *(1) What is the largest prime number less than 100?* [97] *(2) If the digits of the prime number 13 are reversed, 13 becomes 31 which is also a prime number. What is the next prime number that's reversible?* [17, 71] *Can you find 2 more?* [37, 73] [79, 97] *(3) Research information on the Sieve of Eratosthenes.*

22%

4D *<u>Strategy</u>: Find the total number of marbles.*
Since $\frac{2}{5}$ of the marbles have some red on them, the number of marbles is a multiple of 5. Likewise, the number of marbles is a multiple of 4 and of 7. The least common multiple of 4, 5, and 7 is 140. The next common multiple is 280, which is too large. Mia has 140 marbles. $\frac{6}{7}$ of 140 = 120. **120 marbles have some blue on them.**

12%

4E METHOD 1: *<u>Strategy</u>: Use a tree diagram to list each path.*
To simplify the listing, the middle row uses lower case letters and the top row uses circled italicized letters in a different font. Trace each path that starts at **C** and leads to **L**.

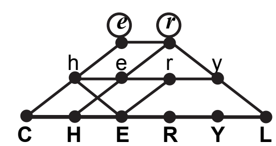
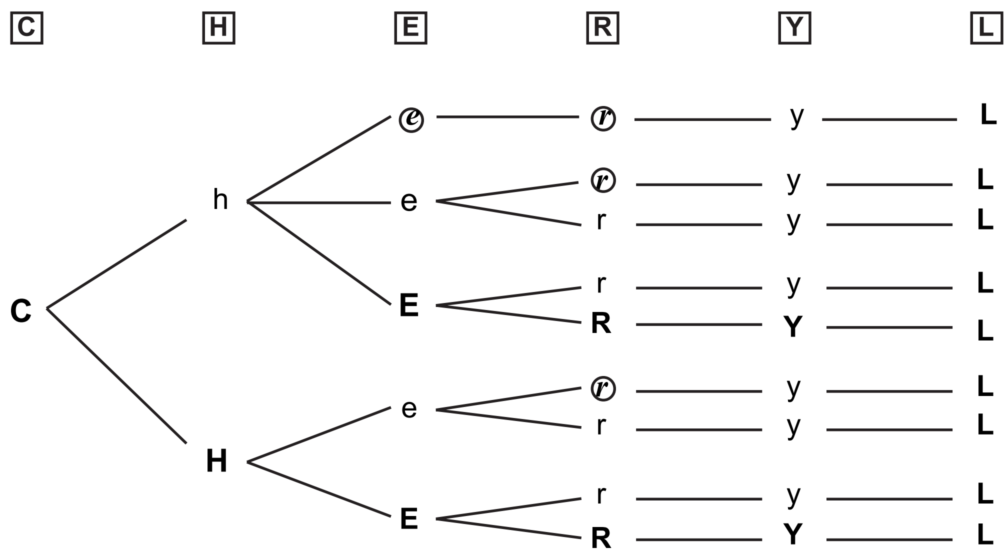


Figure 1



There are 9 paths that trace her name.

METHOD 2: *Strategy: Show the number of ways to reach each point.* Figure 2 is the same as Figure 1 (previous page). Figure 3 shows that there is only one way to reach points **C**, **H**, h, and ⓔ. There are only two ways to reach points **E**, **R**, **Y**, and e. With three ways to reach point ⓡ and four ways to reach point r, there are seven ways to reach point y. Thus, there are 9 ways to reach point **L**.

Figure 2

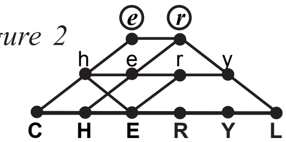
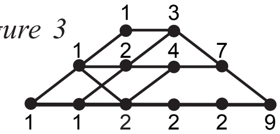


Figure 3



FOLLOW-UP: Using the letters in MATH, how many different 3-letter passwords can be made if no letter is repeated? [24]

OLYMPIAD 5 MARCH 8, 2011

Answers: [5A] **5** [5B] **843** [5C] **18** [5D] **243** [5E] **240**

78% correct

5A *Strategy: Group the factors to simplify the multiplication.*

METHOD 1: (2×3×5) × (2×3×5) × (2×3×5) = 30×30×30 = 27,000. **There are 5 digits in the product.**

METHOD 2: 3×3×3 × (2×5) × (2×5) × (2×5) = 27×10×10×10 = 27,000. There are 5 digits in the product.

FOLLOW-UP: The number 10,000 is written as the product of two numbers, neither of which has 10 as a factor. What is the sum of these two numbers? [641]

61%

5B *Strategy: List the possibilities for each digit.*

The hundreds digit is 4 or 8.

The tens digit could be 1 or 4 or 9. Since the tens digit is less than the hundreds digit, it cannot be 9. The tens digit is 1 or 4.

The ones digit could be 3 or 6 or 9. Again, since the ones digit is less than the tens digit, it cannot be 6 or 9. The ones digit must be 3.

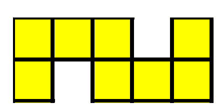
Working backward, since the tens digit must be greater than 3, the tens digit is 4. The hundreds digit is then 8.

Lisa's secret number is 843.

61%

5C *Strategy: Find the length of a side of a small square.*

All eight small squares are congruent, and their total area is 8 square centimeters. Thus, each small square has an area of 1 square centimeter, and each small square is 1 cm by 1 cm. Count the number of small square edges going around the outside the figure. **The perimeter of the figure is 18 cm.**



FOLLOW-UPS: (1) Suppose the perimeter of the given figure is 72 cm. What is its area? [128 sq cm] *(2) Exploration: How many different figures can you form from the 8 original squares? What are their perimeters?* [results will vary.]

11%

5D **METHOD 1:** *Strategy: Break the problem into cases.*
Numbers with exactly two digits the same look like ABB or BAB or BBA.

(a) First consider numbers of the form ABB. If A = 1, there are nine such numbers (100, 122, 133, …, 199). There are also nine such numbers if A = 2, if A = 3, if A = 4, …, and if A = 9.

(b) Consider numbers of the form BAB. If A = 0, there are nine such numbers (101, 202, 303, …, 909). There are only eight such numbers if A = 1, if A = 2, …, and if A = 9, since B may not equal 0. Thus, there are 8 × 9 + 9 = 81 numbers of the form BAB.

(c) A similar argument shows that there are 81 numbers of the form BBA.

Therefore, 3 × 81 = **243 three-digit numbers have exactly two digits the same.**

METHOD 2: *Strategy: Count the numbers that don't have exactly two digits the same.*
The digits of a 3-digit number are in 3 classes. Either all are the same, just two of them are the same, or all are different.

(a) All the same: They are of the form AAA. There are 9 such numbers (111, 222, 333, …, 999).

(b) All different: They are of the form ABC. There are 9 choices for the digit A, since A cannot be 0. For each value of A, there are 9 choices left for B (since B could be 0). For each pair of values of A and B, there are 8 choices for C. Thus, there are 9 × 9 × 8 = 648 numbers of the form ABC.

(c) Since there are 999 numbers less than 1000, of which 99 have two digits, there are 900 three-digit numbers. Then 900 – 648 – 9 = 243 three-digit numbers have exactly two identical digits.

METHOD 3: *Strategy: Make a list.*

Repeated digit	List of numbers	Quantity	
2 zeroes	100, 200, 300, …, 900		9
2 ones	110, 112, 113, …, 119 (not 111)	9	
	101; 121, 131, …, 191 (not 111)	9	26
	211, 311, 411, …, 911 (not 111)	8	
2 twos	220, 221, 223, …, 229 (not 222)	9	
	202; 212, 232, …, 292 (not 222)	9	26
	122, 322, 422, …, 922 (not 222)	8	

As shown in the table, 2 zeroes appear in nine numbers. Also, 2 ones appear in another 26 numbers. The same is true for the 2 twos, 2 threes, …, and 2 nines. This is a total of 9 + (26 × 9) = 243 numbers. In all, there are 243 three-digit numbers with two identical digits.

14%

5E *Strategy: Determine the possible numbers of normal leaves.*
The number of normal clovers is a multiple of 5, since there are 5 times as many normal clovers as lucky clovers. The number of normal clovers is also even, since there are twice as many of them as there are of broken clovers. The number of normal clovers is therefore a multiple of 10.

For each 10 normal clovers, there are 2 lucky clovers and 5 broken clovers.

Split the collection of N clovers into groups so that each group has 10 normals, 2 luckies, and 5 brokens. The number of leaves in each group is 10×3 + 2×4 +5×2 = 48.

The total number of leaves is a multiple of 48, and the least multiple of 48 that is greater than 200 is 5×48 = 240. **The least value of N is 240.**