2014 TOURNAMENT

Grade 6 and Below

PRACTICE PROBLEMS FOR 2016

Table of Contents

2014 Individual Event

No calculators are permitted during this tournament.
Time Limit: 30 minutes.

1. What number belongs in the box to make the statement true? $11 \times \boxed{} = 4321 \times 22$

2. Mary had 20 little lambs. All but 6 ran away. How many little lambs did Mary then have?

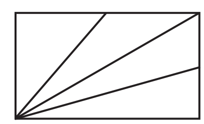
3. In the multiplication example at the right, A and B represent two different digits. What two-digit number does AB represent?

$$\begin{array}{r} 3\ 6\ \mathrm{A} \\ \times\quad \mathrm{A}\,\mathrm{B} \\ \hline 3\ 6\ \mathrm{A}\,\mathrm{B} \end{array}$$

4. A restaurant has 5 tables that seat 2 people each, 10 tables that seat 4, and 3 tables that seat 6. Each waitress can handle up to 15 people. What is the fewest number of waitresses needed if every seat is taken?

5. How many odd numbers between 10 and 100 are divisible by 3?

6. How many acute angles of all sizes appear in the rectangle at the right?



7. The meaning of 2! is 2 × 1, the meaning of 3! is 3 × 2 × 1, and the meaning of 4! is 4 × 3 × 2 × 1. Their values are 2, 6, and 24 respectively. What whole number must be subtracted from 5! in order to result in the largest perfect square number that is less than 5!?

8. A poster contains two rectangular photographs, *ABCD* and *JKLM*. On that poster *AB* = 40 mm, *BC* = 30 mm, *JK* = 28 mm and *KL* = 24 mm. If the poster and everything on it is enlarged uniformly so that *AB* becomes 50 mm, what is the area of the enlarged rectangular photograph *JKLM*?

9. What is the value of 2500 + 2499 – 2498 – 2497 + 2496 + 2495 – 2494 – 2493 + … + 4 + 3 –2 – 1, if the pattern of the addition of two consecutive integers alternating with the subtraction of two consecutive integers is followed throughout?

10. The fraction X can be called an extended complex fraction. What simple fraction in lowest terms is equivalent to X?

$$X = \cfrac{1}{2 + \cfrac{3}{4 + \frac{5}{6}}}$$

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SCORE

Student Name ____________________

School and Team ____________________ Team # __________

2014 Individual Event: Answers

Write answers clearly. Each correct answer will receive one point.

1. ____________________
2. ____________________ **little lambs**
3. **AB =** ____________________
4. ____________________ **waitresses**
5. ____________________ **numbers**
6. ____________________ **angles**
7. ____________________
8. ____________________ **sq mm**
9. ____________________
10. ____________________

2014 Team Event

No calculators are permitted during this tournament.
Time limit: 20 minutes

11. What is the least multiple of 12 that is greater than 200?

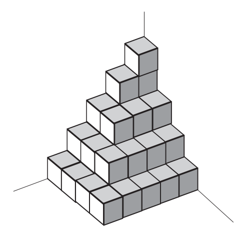
12. What is the sum of the prime factors of 105?

13. The math club has 22 members. The school band has a total of 26 members. A total of 41 people have joined at least one of these activities. How many people have joined both activities?

14. Toni has a total of 34 pennies and nickels. Their total value is 98 cents. How many pennies does she have?

15. Every digit of a 100-digit number is a 7. What is the remainder if the number is divided by 12?

16. The tower shown consists of cubes piled on each other. There are no gaps. After the tower is painted on all sides including the bottom and backs, it is cut into individual cubes. How many cubes have been painted on exactly 3 faces?



17. The numerator of a fraction contains two of the numbers 2, 3, 5, 7, and 9, which are added, subtracted, or multiplied. The denominator contains two of the other numbers and they are added, subtracted, or multiplied. What is the greatest possible value of the fraction?

18. How many different sums are possible if ten standard dice are rolled?

19. A rectangular piece of tin is 9 units wide and 12 units long. A square 2 units on each side is cut from each corner and discarded. The four flaps of the remaining figure are then folded up to form a tin box with no top in the shape of a rectangular solid. What is the volume of the resulting rectangular solid?

20. The cost for the computer club to replace its laptop is $315, to be shared equally by all members. The school will pay the rest. By recruiting four new members to join the club, the cost to each member dropped by $5. What was the cost per member before the four new members joined the club?

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School and Team ____________________ Team # ________

Student Names ______________________________

SCORE

2014 Team Event: Answers

Write answers clearly. Each correct answer will receive one point.

11. ____________________

12. ____________________

13. ____________________ people

14. ____________________ pennies

15. ____________________

16. ____________________ cubes

17. ____________________

18. ____________________ sums

19. ____________________ cubic units

20. $ ____________________

SCORING ROOM KEY

2014

INDIVIDUAL Event: Answers		TEAM Event: Answers	
1.	8642	11.	204
2.	6	12.	15
3.	10	13.	7
4.	5	14.	18
5.	15	15.	1
6.	13	16.	13
7.	20	17.	63
8.	1050	18.	51
9.	2500	19.	80
10.	$\frac{29}{76}$	20.	$22.50

INDIVIDUAL EVENT SOLUTIONS, 2014

ANSWERS: 1) **8642** 2) **6** 3) **10** 4) **5** 5) **15**
6) **13** 7) **20** 8) **1050** 9) **2500** 10) $\frac{29}{76}$

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1. ***METHOD 1:*** Notice that 22 = 2 × 11. Then 4321 × 22 = 4321 × 2 × 11 = 8642 × 11. [*Note: This actually uses the associative property: 4321 × (2 × 11) = (4321 × 2) × 11*]

   METHOD 2: 4321 × 22 = 95062. Then 95062 ÷ 11 = 8642.

2. All ***but*** 6 ran away, so Mary had those 6 lambs left.

3. Notice that the first 3 digits of the product, 36A, are the same as those of the multiplicand. When all numbers are whole, this happens only when the multiplier is 10.

$$\begin{array}{r} 3\ 6\ A \\ \times \quad A\ B \\ \hline 3\ 6\ A\ B \end{array} \qquad \begin{array}{r} 3\ 6\ 1 \\ \times \quad 1\ 0 \\ \hline 3\ 6\ 1\ 0 \end{array}$$

4. Divide the total number of diners by 15. Since 68 ÷ 15 = 4R8, 4 waitresses can handle up to 15 diners each. The remaining 8 diners must be handled by a 5th waitress.

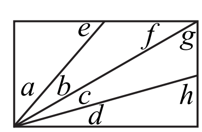
# of tables	# of seats per table	# of diners
5	2	10
10	4	40
3	6	18
	Total number of diners	68

5. ***METHOD 1:*** Of the 90 numbers between 10 and 100, 30 are divisible by 3. Of these, the 15 even numbers are divisible by 2 and 15 odd numbers are not divisible by 2.

   METHOD 2: List all multiples of 3 between 10 and 100. Then cross out all even multiples on the list. The 15 numbers that are left are 15, 21, 27, 33, 39, 45, 51, 57, 63, 69, 75, 81, 87, 93, and 99.

6. Label as shown the 8 individual angles which are acute. The unlabeled angles are either right angles or obtuse angles. Then list separately those acute angles which contain 1, 2, or 3 individual acute angles.

acute angles by number of individual angles		number of angles
1 such angle:	*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*	8
2 such angles:	*ab*, *bc*, *cd*	3
3 such angles:	*abc*, *bcd*	2
	total =	13



7. Because 5! = 5 × 4 × 3 × 2 × 1, its value is 120. 120 is between the perfect squares 100 and 121 (10 × 10 and 11 × 11). Thus 20 must be subtracted from 120 to result in the largest perfect square number that is less than 5!.

8. **METHOD 1:** AB is enlarged from 40 mm to 50 mm and the change is uniform for every line segment in the photograph. This means that for every 4 mm in the original length of any segment, there now is 5 mm in the length of the enlarged segment. Thus, JK has changed from $28 = 4 \times 7$ mm to $5 \times 7 = 35$ mm and KL has changed from $24 = 4 \times 6$ mm to $5 \times 6 = 30$ mm. The area of the enlarged rectangle $JKLM$ is $35 \times 30 = \boxed{1050}$ sq mm.

METHOD 2: The ratio of the "old" AB to the "new" AB is 40:50, or 4:5. Let x and y be the lengths of the "new" JK and KL respectively in millimeters. Then $\frac{4}{5} = \frac{28}{x}$ and $\frac{4}{5} = \frac{24}{y}$. Solving each proportion, $x = 35$ and $y = 30$. The area of the enlarged rectangle $JKLM$ is $35 \times 30 = 1050$ sq mm.

9. The value of every group of 4 terms is 4. For example, $(2500 - 2498) + (2499 - 2497) = 4$ and $(4 - 2) + (3 - 1) = 4$. Then 2500 terms is equivalent to a value of 4 that occurs 625 times for a total of $\boxed{2500}$.

10. (1) Change $4 + \frac{5}{6}$ to an improper fraction: $\frac{6 \times 4 + 5}{6} = \frac{29}{6}$

(2) Simplify $\frac{3}{4+\frac{5}{6}}$: $\frac{3}{\frac{29}{6}} = 3 \div \frac{29}{6} = 3 \times \frac{6}{29} = \frac{18}{29}$

(3) Now change $2 + \frac{3}{4+\frac{5}{6}}$ to an improper fraction: $2 + \frac{3}{4+\frac{5}{6}} = 2 + \frac{18}{29} = \frac{29 \times 2 + 18}{29} = \frac{76}{29}$

(4) Lastly, simplify $\frac{1}{2+\frac{3}{4+\frac{5}{6}}}$: $\frac{1}{\frac{76}{29}} = 1 \div \frac{76}{29} = 1 \times \frac{29}{76} = \boxed{\frac{29}{76}}$

TEAM EVENT SOLUTIONS, 2014

ANSWERS:					
	11) **204**	12) **15**	13) **7**	14) **18**	15) **1**
	16) **13**	17) **63**	18) **51**	19) **80**	20) **$22.50**

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11. ***METHOD 1:*** 200 ÷ 12 = 16R8. With a remainder of 8, add 4 to get to the next multiple of 12, which is **204**.

    METHOD 2: Any multiple of 12 is also a multiple of both 4 and 3. Use tests of divisibility to discover that 200 is a multiple of 4, but not of 3. The next few multiples of 4 are 204, 208, 212, and 216. The sum of the digits of 204 is a multiple of 3. Therefore, 204 is the least multiple of 12 that is greater than 200.

    METHOD 3: Divide each whole number greater than 200 by 12 until you get a remainder of zero.

Numbers over 200	201	202	203	**204**
Remainder	9	10	11	**0**

The multiple of 12 is 204.

12. 105 = 5 × 21 = 5 × 3 × 7. There are no other prime factors. The sum of 3, 5, and 7 is **15**.

13. 22 people join the math club. 26 people join the school band. Thus, a total of 48 people seemed to join one of the two activities. But there are only 41 people. Therefore 48 – 41 = **7** people joined both activities.

14. ***METHOD 1:*** The value of 34 pennies is only 34¢, so there must be mostly nickels. With a total of 98¢, there are fewer than 20 nickels. Starting with 19 nickels, make an organized table of nickels and pennies with a total value of 98¢. Look for a total of 34 coins.

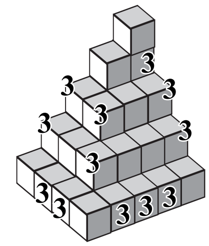
Number of nickels	19	18	17	**16**	Count down from 19 nickels.
Value of nickels	95¢	90¢	85¢	**80¢**	Multiply row 1 by 5¢.
Value of pennies	3¢	8¢	13¢	**18¢**	Subtract row 2 from 98¢.
Number of coins	22	26	30	**34**	Toni has **18** pennies.

METHOD 2: A similar table can be used in which the total number of coins is always 34 and you look for a total value of 98¢.

15. Start with a simpler related problem and look for a pattern: divide 7,777, 777, ... , 777 by 12 and examine each remainder. From left to right they are 5, 9, 1, 5, 9, 1, repeating endlessly. Every 3-digit period (thousands, millions, billions, etc.) starts with a remainder of 1 from the previous period. The remainder after the ones digit will be **1**.

$$\begin{array}{r} 6\,4\,8,\ 1\,4\,8,\ 1\,4\,8,\ \ldots,\ 1\,4\,8 \\ 12\overline{\big)\,7,7\,{}^{5}7\,{}^{9}7,\ {}^{1}7\,{}^{5}7\,{}^{9}7,\ {}^{1}7\,{}^{5}7\,{}^{9}7,\ \ldots,\ {}^{1}7\,{}^{5}7\,{}^{9}7} \end{array}$$

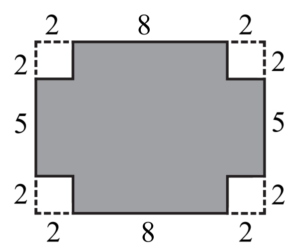
16. Only the surface cubes, including those on the bottom, have paint on any faces. The 12 cubes that show the number "3" each have 3 painted faces. In some cases the unseen face is in the back or on the bottom. There is one more cube in the back of the tower at the bottom that has 3 painted faces. In all, a total of **13** cubes have exactly 3 painted faces. All the rest have 0, 1, 2, 4, or 5 painted faces.

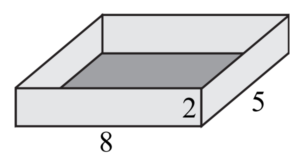


17. To maximize the fraction, maximize the numerator and minimize the denominator. The greatest value of the numerator is produced by multiplying the two largest numbers together: $9 \times 7 = 63$. The least value of the denominator is produced by subtracting the two closest numbers: $3 - 2 = 1$. The value of the fraction is $\frac{63}{1}$ or **63**.

18. The least sum occurs if each of the ten dice show a 1. That sum is 10.
The greatest sum occurs if each of the ten dice show a 6. That sum is 60.
By changing one or more dice, every sum from 10 to 60 inclusive can be obtained. Thus, every sum from 1 to 60 inclusive is possible except for the sums from 1 to 9. There are $60 - 9 =$ **51** possible sums.

19. As the diagram on the left shows, the 2-unit squares are cut from each corner, and the lengths of the remaining edges are 5 units and 8 units. The volume of the rectangular solid (on the right) is the product of its length, width and height. The volume is $8 \times 5 \times 2 =$ **80** cubic units.





20. A table is very helpful. List some possible numbers of members (since they are whole numbers) and the shares to be paid by each.

Number of members	1	2	3	4
Cost per members	\$315	\$157.50	\$105	\$78.75

Notice that the table suggests two things. There is always a whole number of members, but there doesn't have to be a whole number of dollars in each member's cost. And as the number of members increases by only one, each member's share decreases by a large amount. This suggests that the two numbers should be close together and that the number of members is much larger. Try some larger members. Discard any results that are not a whole number of cents.

Number of members	10	11	12	13	**14**	15	16	17	**18**
Cost per members	\$31.50	—	\$26.25	—	**\$22.50**	\$21	—	—	**\$17.50**

The only places in the table where the costs drop by \$5 as the number of members increase by 4 is from the first highlighted box to the second one. Before the new members joined, each member expected to pay **\$22.50**.

TIEBREAKER #1

Time Limit: 5 minutes.
No calculators are permitted.

Name ________________________________ Team _________________________

Two different prime numbers are chosen from the first five prime numbers. What is the probability that their sum is prime?

Answer _____________________

TIEBREAKER #2

Time Limit: 5 minutes.
No calculators are permitted.

Name ______________________________ Team ________________________

Three friends split the cost equally of a whole apple pie. Each pays \$2.50. Suppose two more friends had joined them. How many dollars would each then pay?

Answer ____**\$**______**.**__________

TIEBREAKER #3

Time Limit: 5 minutes.
No calculators are permitted.

Name ________________________________ Team _________________________

Two counting numbers have a sum of 7 and are multiplied. What is the sum of all of the possible products?

Answer _____________________

TIEBREAKER #4

Time Limit: 5 minutes.
No calculators are permitted.

Name ________________________________ Team _________________________

Linda starts with the number two and then counts by fours.
Her first five numbers are 2, 6, 10, 14, and 18.
What is her 30^{th} number?

Answer _____________________

TIEBREAKER #5

Time Limit: 5 minutes.
No calculators are permitted.

Name ________________________________ Team _________________________

The digits of a four-digit number are 4, 5, 6, and 7, but not in that order. It is a multiple of 25 and it is less than 6000. What is the four-digit number?

Answer _____________________

2014 TIEBREAKERS

Questions are given one at a time. Winning places are awarded in the order that correct answers are submitted. Incorrect answers result in elimination. No calculators are permitted during this tournament. Time limit: 5 minutes per question.

T1. Two different prime numbers are chosen from the first five prime numbers. What is the probability that their sum is prime?

T2. Three friends split the cost equally of a whole apple pie. Each pays \$2.50. Suppose two more friends had joined them. How many dollars would each then pay?

T3. Two counting numbers have a sum of 7 and are multiplied. What is the sum of all of the possible products?

T4. Linda starts with the number 2 and then counts by 4s. Her first five numbers are 2, 6, 10, 14, and 18. What is her 30th number?

T5. The digits of a four-digit number are 4, 5, 6, and 7, but not in that order. It is a multiple of 25 and it is less than 6000. What is the four-digit number?

2014 TIEBREAKER SOLUTIONS

ANSWERS: T1) $\frac{3}{10}$ T2) **\$1.50** T3) **28** T4) **118** T5) **4675**

T1. The first five prime numbers are 2, 3, 5, 7, and 11.
There are ten possible sums of any two numbers: 5, 7, 9, 13; 8, 10, 14; 12, 16; and 18.
Of these, only three sums are prime: 5, 7, and 13.
The probability that their sum is prime is $\frac{3}{10}$.

T2. The whole pie costs 3 × \$2.50 = \$7.50. Split five ways, each friend would pay \$7.50 ÷ 5 = **\$1.50**.

T3. Notice that 3 × 4 and 4 × 3 have the same product. There are 3 products: 1 × 6 = 6, 2 × 5 = 10, and 3 × 4 = 12. Then 6 + 10 + 12 = **28**.

T4. ***METHOD 1:*** With 30 numbers, there are 29 intervals. Thus the 30th number is 2 + 29 × 4 = **118**.

***METHOD 2*:** Compare the first 5 numbers with the first 5 non-zero multiples of 4: 4, 8, 12, 16, 20. Each of Linda's numbers is 2 less than the corresponding multiple. Then the 30th such multiple is 4 × 30 = 120, so her 30th number is 118.

***METHOD 3*:** Make a list of all 30 numbers. Listing five numbers at a time helps you keep track.

T5. Any multiple of 25 ends in 00, 25, 50, or 75, so the four-digit number is either 4675 or 6475. Since it is less than 6000, the number is **4675**.